INVESTIGACIONES ECONÓMICAS. vol. XXXII (3), 2008, 325-338

ON ENDOGENOUS CARTEL SIZE UNDER TACIT COLLUSION

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We analyze how the size of a cartel affects the possibility to sustain a collusive agreement. We develop a multi-period oligopoly model with homogeneous, quantity-setting firms, a subset of which are assumed to collude, while the remaining (fringe) firms choose their output levels noncooperatively. We show that, in our model, collusion is easier to sustain the larger the cartel is. The implications of this result on the incentives of firms to participate in a cartel are analyzed. We obtain that a firm is only willing to collude when otherwise collusion cannot be sustained.

Keywords: Collusion, partial cartels, trigger strategies, optimal punishment.

(*JEL* L11, L13, L41, D43)

1. Introduction

Looking at the most recent measures of the European Commission against collusion, we see that several firms have been fined hundreds millions euros for price fixing and setting sales quotas. It can also be observed that, in practice, many collusive agreements do not involve all firms in the industry.[1] Despite the empirical evidence, the Industrial Organization analysis of implicit collusion in quantity-setting supergames has usually focused on the symmetric subgame perfect Nash

This paper is based on Chapter 1 of my PhD thesis at the University of Alicante (Spain). I am grateful to my supervisor Ramon Faulí-Oller for his advice and encouragement. Earlier versions of this paper were presented at SMYE, Alicante, Frankfurt, EARIE and ASSET conferences. Financial support by the I.V.I.E. *(Instituto Valenciano de Investigaciones Económicas)*, Ministerio de Ciencia y Tecnologia through its project Gobierno Corporativo (Ref: SEJ2005-08462) and the European Commission through its RTN programme (Research Training Network) is gratefully acknowledged. Any remaining errors are my own.

[1] As an example, three North-American and five European firms in the citric acid industry were fined for fixing prices and allocating sales in the worldwide market. Their joint market share was around 60 percent (see Levenstein and Suslow (2006)).

equilibrium —henceforth, SPNE— that maximizes industry profits (see for example the seminal paper by Friedman (1971) or Rothschild (1999)). In such models, it is assumed that seemingly independent, but parallel actions among competing firms in an industry are driven to achieve higher profits. However, this approach focuses on firms' *incentive constraints* and leaves out firms' *participation constraints.*[2] In other words, it cannot explain why many real world cartels do not comprise all firms in the industry.

The main aim of the present paper is twofold 1) we study how the number of firms in a cartel affects the possibility that its members can sustain a collusive agreement, and 2) using the concept proposed by d' Aspremont *et al.* (1983) for cartel stability, we endogeneize cartel formation by analyzing the number of firms that are willing to participate in a cartel when firms have the possibility to implicitly collude.

We develop a multi-period oligopoly model with homogeneous, quantity-setting firms, a subset of which are assumed to collude, while the remaining (fringe) firms choose their output levels noncooperatively. We use SPNE as solution concept. It is well known that our repeated game setting exhibits multiple SPNE collusive agreements. Therefore, to select among those equilibria we adopt the particular criterion of 1) restricting strategies to grim *trigger strategies,* and 2) adding an initial stage to the game where firms simultaneously choose whether or not to join the (unique) cartel.

We obtain that, in our model, collusion is easier to sustain the larger the cartel is. This result stems from the fact that an increase in the size of the cartel has the effect of increasing the profits of a cartel firm and the profits attained by an optimal deviation from the collusive agreement, while not modyfing the Cournot profits. Hence, if the collusive agreement is maintained by means of the threat of Cournot reversion in the event that deviations occurs, then firms face a harsher punishment when the size of the cartel increases.

This result parallels those on the literature on mergers where strategic incentives to merge are positive for a sufficiently large number of merging firms (see for example Salant, Switzer and Reynolds (1983)

[2] *Participation constraints* determine the incentives that firms have to join a cartel. However, *incentive constraints* determine the incentives of cartel firms to deviate from the collusive agreement.

or Perry and Porter (1985)). However, the main contribution of the present paper is that this result has crucial implications on cartel formation. The common intuition suggests that although there is a general firms' interest in the existence of a cartel, the benefits of cartel formation are not evenly distributed and often nonmembers obtain higher profits than cartel members.[3] We obtain that firms are willing to participate in the smallest cartel among those which can be sustained as a SPNE because although firms might have incentives to exit the cartel, the threat of no collusion at equilibrium induces them to collude. In other words, only the smallest cartel is stable because although firms might have incentives to exit the cartel, if they do it collusion collapses and cartel firms would be worse-off. We can illustrate this result with the following extreme example. We find that for some values of the discount factor, the only sustainable cartel is the cartel that comprises all firms in the industry. Then, all firms have incentives to participate in the cartel because otherwise no collusion is possible. This completely eliminates the gains from free-riding at the participation stage. Our results extend the results on cartel stability by Selten (1973), d'Aspremont *et al.* (1983), Donsimoni (1986) and Shaffer (1995) to the case of implicit collusion.[4]

The remainder of the paper is organized as follows. In Section 2, we present the model. In Section 3, we analyze the cartel stability. Section 4 tests the robustness of our results. We obtain that our results are robust to 1) using an optimal punishment —the *stick-and-carrot strategies* proposed by Abreu (1986,1988)— and 2) the choice of a convex or a linear cost function. However, in line with the literature on mergers our results are not robust to assuming price competition (see for example Davidson and Deneckere (1985)). We conclude in Section 5. All proofs are grouped together in the appendix.

2. The model

We consider an industry with $N > 2$ firms, indexed by $i = 1, ..., N$. Each firm produces a quantity of a homogeneous product with a quadra-

[3] The intrinsic difficulty in convincing firms to constitute a cartel was pointed out by Stigler (1950, p. 25) in a discussion of mergers: "The major difficulty in forming a merger is that it is more profitable to be outside a merger than to be a participant".
[4] A one-stage game is assumed in those papers. Then, using the concept proposed by d'Aspremont *et al.* (1983) for cartel stability, they obtain that generally only cartels containing just over half the firms in the industry are stable.

tic cost function $c(q_i) = cq_i^2$, where q_i is the output produced by firm i. We assume that firms simultaneously choose quantities. The industry inverse demand is given by the piecewise linear function

$$p(Q) = \max(0, a - Q),$$

where $Q = \sum_{i=1}^{N} q_i$ is the industry output, p is the output price. We assume that one cartel is formed, in such a way that $K \leq N$ firms, indexed by $k = 1, ..., K$ —henceforth, cartel firms— behave cooperatively so as to maximize their joint profits. The remaining $(N - K)$ firms constitute the fringe and choose their output in a non-cooperative way.

We assume that firms compete repeatedly over an infinite horizon with complete information (i.e. each of the firms either fringe or cartel observes the whole history of actions) and discount the future using a discount factor $\delta \in (0, 1)$. Time is discrete and dates are denoted by $t = 1, 2, ...$ In this framework, a pure strategy for firm k is an infinite sequence of functions $\left\{S_k^t\right\}_{t=1}^{\infty}$ with $S_k^t : \sum^{t-1} \longrightarrow \mathcal{Q}$ where $\sum^{t-1}$ is the set of all possible histories of actions (output choices) of all cartel firms up to $t - 1$, with typical element σ_j^τ, $j = 1, ..., K$, $\tau = 1, ..., t - 1$, and $\mathcal{Q}$ is the set of output choices available to each cartel firm. Following Friedman (1971), we restrict our attention to the case where each cartel firm is only allowed to follow grim trigger strategies. In words, these strategies are such that cartel firms adhere to the collusive agreement until there is a defection, in which case they revert forever to the static N-firm Cournot equilibrium. Let q and q_n denote the output corresponding respectively to collusion and Cournot noncooperative behavior. Since we restrict attention to trigger strategies, $\left\{S_k^t\right\}_{t=1}^{\infty}$ can be specified as follows. At $t = 1$, $S_k^1 = q$, while at $t = 2, 3, ...$

$$S_k^t(\sigma_j^\tau) = \begin{cases} q \text{ if } \sigma_j^\tau = q \text{ for all } j = 1, ..., K, \ \tau = 1, ..., t-1 \\ q_n \text{ otherwise.} \end{cases} \quad [1]$$

Regarding fringe firms, their optimal response consists of maximizing their current period's payoff. We denote by q_f the output produced by each fringe firm. It can be verified that when K firms behave cooperatively so as to maximize their joint profits, then the equilibrium quantities are

$$q = \frac{a(1 + 2c)}{K(N - K + 2) + 4c^2 + 2c(N + K + 1)} \quad [2]$$

$$q_f = \frac{a(K + 2c)}{K(N - K + 2) + 4c^2 + 2c(N + K + 1)}.$$

We denote the profit function of a cartel firm and that of a fringe firm by $\Pi^c(N,K)$ and $\Pi^f(N,K)$ respectively. We note that $\Pi^c(N,K)$ and $\Pi^f(N,K)$ decrease with N. On the other hand, $\Pi^f(N,K)$ increases with K, and so does $\Pi^c(N,K)$ except when the cartel is very small.[5]

If a cartel firm deviates from q it produces $q_d = \arg\max_x P((K-1)q + (N-K)q_f + x)x - cx^2$ and obtains profits that we denote by $\Pi^d(N,K)$. As shown by Friedman (1971), cartel firms producing q in each period can be sustained as a SPNE of the repeated game with the strategy profile [1] if and only if for given values of N, K and δ, the following condition is satisfied

$$\frac{\Pi^c(N,K)}{1-\delta} \geq \Pi^d(N,K) + \frac{\delta \Pi(N)}{1-\delta} \qquad [3]$$

where $\Pi(N)$ denotes the Cournot equilibrium profits. For given values of N and K, we denote by δ_K the minimum δ required for the condition [3] to be satisfied.

DEFINITION 1 *δ_K is said to be the minimum discount factor required for the cartel of K firms to be sustainable as a SPNE. Then, a cartel of size K is said to be sustainable if $\delta \geq \delta_K$ and $\delta_K \in (0,1)$.*

We can now state the following Proposition.

PROPOSITION 1 *δ_K is strictly decreasing in K.*

Proposition 1 establishes that collusion is easier to sustain the larger the cartel is.[6] We note that δ_K can be rewritten like

$$\delta_K = \frac{1 - \dfrac{\Pi^c(N,K)}{\Pi^d(N,K)}}{1 - \dfrac{\Pi(N)}{\Pi^d(N,K)}}.$$

Then, variations of K have two different effects. First, $\dfrac{\Pi^c(N,K)}{\Pi^d(N,K)}$ decreases with K since deviation profits increase more than cartel profits.

[5] In particular, $\Pi^c(N,K)$ increases with K when $K > \frac{2(1-c)+n+\sqrt{24c+(2(1-c)+n)^2}}{6}$ and decreases otherwise. We note also that $\Pi^f(N,K) > \Pi^c(N,K)$ since fringe firms gain more than cartel members from the high price induced by the cartel.
[6] We note the similarity to the result by Salant et al. (1983) where if a merger of K firms is profitable, a merger involving more than K firms is also profitable.

Second, $\frac{\Pi(N)}{\Pi^d(N,K)}$ also decreases with K given that $\Pi(N)$ does not depend on K and deviation profits increase with K. Proposition 1 comes from the fact that the second effect dominates the first one. Intuitively, for given values of N, as K increases cartel firms face a harsher punishment in the event that deviation from the collusive agreement occurs, and thus collusion is easier to sustain.

3. The participation game

We add in this section an initial stage to the game we have considered so far, in which firms simultaneously choose whether or not to join the (unique) cartel. We denote by $\Pi_0^c(N,K)$ and $\Pi_0^f(N,K)$ the discounted profits attained by cartel and fringe firms respectively in the initial stage

$$\Pi_0^c(N,K) = \begin{cases} \frac{1}{1-\delta}\Pi^c(N,K) \text{ if } \delta \geq \delta_K \\ \frac{1}{1-\delta}\Pi(N) \quad \text{otherwise} \end{cases}$$

$$\Pi_0^f(N,K) = \begin{cases} \frac{1}{1-\delta}\Pi^f(N,K) \text{ if } \delta \geq \delta_K \\ \frac{1}{1-\delta}\Pi(N) \quad \text{otherwise.} \end{cases}$$

It is well known that a repeated game setting exhibits multiple SPNE collusive agreements. Therefore, to select among those equilibria, we adopt the particular criterion of restricting attention to the strict Nash equilibria. At this initial stage, a strict Nash equilibrium is defined as follows: 1) a number $0 < K < N$ of firms join the cartel if and only if

$$\Pi_0^c(N,K) > \Pi_0^f(N,K-1) \qquad [4]$$

and

$$\Pi_0^f(N,K) > \Pi_0^c(N,K+1) \qquad [5]$$

2) all N firms join the cartel if and only if

$$\Pi_0^c(N,N) > \Pi_0^f(N,N-1) \qquad [6]$$

3) no firm joins the cartel if and only if

$$\Pi_0^f(N,0) > \Pi_0^c(N,1). \qquad [7]$$

Using the terminology in d'Aspremont *et al.* (1983), conditions [4] and [6] guarantee the internal stability of the cartel (no firm inside the cartel finds it desirable to exit), while conditions [5] and [7] guarantee the external stability (no firm outside the cartel finds it desirable to enter).[7]

DEFINITION 2 *A cartel is said to be stable if it is both internally and externally stable.*

We are now in the position to obtain the strict Nash equilibrium of the repeated game with an initial participation stage, which is to determine the number of firms in the stable cartels. As mentioned earlier, this setting exhibits multiple equilibria. However, we focus on the equilibria where a number $0 < K \leq N$ of firms join the cartel.[8] We obtain:

PROPOSITION 2 *For given values of N, a cartel of K firms is stable whenever $\delta_K \leq \delta < \delta_{K-1}$ and $\delta_K < 1$.*

The intuition behind Proposition 2 is as follows. If $\delta_K \leq \delta < \delta_{K-1}$ only a cartel of K or more firms is sustainable. In this case, a cartel of more than K firms is not stable because firms inside the cartel find it desirable to exit. However, a cartel of K firms is stable because by leaving the cartel a firm would obtain lower profits (namely, the Cournot profits) since the cartel of $K - 1$ firms is not sustainable. In other words, only the smallest cartel among those that are sustainable is stable in the participation game, because although firms have incentives to exit the cartel, the threat of no collusion at equilibrium induces them to collude.

4. Robustness of results

4.1 Price competition

To test whether our results hinge on the assumption of quantity competition, we study the case of price competition. In order to avoid the Bertrand paradox, we follow the seminal paper by d'Aspremont et al. (1983), and we assume that the cartel acts as price leader, whereas all firms in the fringe are price takers. Members of the fringe take the

[7]We also assume that firms hypothesize that no other firm will change its strategy concerning its membership in the cartel.

[8]We focus on the *strict* Nash equilibria since the analysis of the Nash equilibria would lead us to uninteresting results. For instance, when for each K , $\delta < \delta_K$, then firms obtain the same profits by joining the cartel than by staying at the fringe. Therefore, joining the cartel is always a Nash equilibrium. We note also that the equilibria in which all firms decide not to join the cartel are not considered.

price as given by the cartel and choose their production level by setting marginal cost equal to the price.[9] It can be verified that, contrary to Proposition 1, the critical discount factor is increasing in $.K$, and therefore, the implications on cartel stability derived in Proposition 2 do not carry over when firms compete in prices. This difference follows from the fact that reaction functions are upward sloping in price games but downward sloping in quantity games. Then, the reaction of fringe firms reinforces the initial price increase that results from an increase in K, and deviation profits decrease with K. Therefore, the intuitions provided in Sections 2 and 3 are reversed when firms compete in prices.

4.2 Optimal punishment

It is natural to consider a set of strategies that are less grim than the trigger strategies. We consider here the two-phase output path (with a stick-and-carrot pattern) presented by Abreu (1986, 1988). As in section 2, the strategy space consists of a sequence of decisions rules, describing each player's action as a function of the past history of the play. Then, a pure strategy for firm k is an infinite sequence of functions $\left\{S_k^t\right\}_{t=1}^{\infty}$ with $S_k^t : \sum^{t-1} \longrightarrow \mathcal{Q}$ where $\sum^{t-1}$ is the set of all possible histories of actions of all firms up to $t-1$, with typical element σ_j^{τ}, and $\mathcal{Q}$ is the set of output choices available to each firm. Following Abreu (1986, 1988), we restrict our attention to the case where each firm is only allowed to follow a stick-and-carrot strategy. In other words, we assume that if a deviation from the collusive agreement occurs, then all firms expand their output for one period so as to drive price below cost and return to the most collusive sustainable output in the remaining periods, provided that every player went along with the first phase of the punishment. Let q and q^p denote the output produced by each firm in a collusive and in a punishment phase respectively. $\left\{S_k^t\right\}_{t=1}^{\infty}$ can be specified as follows. At $t = 1$, $S_k^1 = q$, while at $t = 2, 3, ...$

$$S_k^t(\sigma_j^{\tau}) = \begin{cases} q \text{ if } \sigma_j^{\tau} = q \text{ for all } j = 1, ..., K,\ \tau = 2, ..., t-1 \\ q \text{ if } \sigma_j^{\tau} = q^p \text{ for all } j = 1, ..., K,\ \tau = t-1 \\ q^p \text{ otherwise.} \end{cases} \qquad [8]$$

Under the conditions specified in Abreu (1986), each cartel firm producing the quantity q specified in [2] in each period can be sustained as

[9] We note that if fringe firms were not competitive, they would repeatedly undercut cartel firms to steal the market and no cartel would be formed.

a SPNE of the repeated game with the strategy profile [8]. As in section 2, let δ_K denote the minimum discount factor required for a cartel of K firms maximizing their joint profits at equilibrium. Without loss of generality, we consider $a = 1$ and $c = \frac{1}{2}$.

PROPOSITION 3 *δ_K is strictly decreasing in K if $K \leq \min\{N, f(N)\}$ where $f(N) = \frac{13+3N+\sqrt{(9N^2+138N+249)}}{10}$. Otherwise, it is strictly increasing in K.*

We observe that, in this case, δ_K is always decreasing in K only if $N \leq 8$ (namely, if $N < f(N)$).[10] However, a similar result to Proposition 2 is obtained as far as the strict Nash equilibrium of the participation game is concerned.

PROPOSITION 4 *For given values of N, the cartel of 2 firms is not stable. However, if $K > 2$ a cartel of K firms is stable whenever $\delta_K \leq \delta < \delta_{K-1}$ and $\delta_K < 1$.*

The intuition behind the first part of Proposition 4 is as follows. The cartel of 2 firms is not stable since by leaving the cartel a firm would always obtain higher profits. The second part of Proposition 4 parallels Proposition 2, and states that only the smallest cartel among those which can be sustained is stable.

4.3 Linear cost functions

It is well-known that increasing marginal costs soften competition. An important question is whether our results hinge on the assumption of a convex cost function. We study here the case of a linear cost function (namely, $c(q_i) = cq_i$). It can be verified that $\delta_K = \frac{(K-1)(1+N)^2}{4K^2(3+2N)-3K(1+N)^2-(1+N)^2-4K^3}$ and $\frac{\partial \delta_K}{\partial K} < 0$. Thus, with a linear cost function collusion is also easier to sustain the larger the cartel is and the main results of the paper continue to hold. We note that cooperation within a cartel is equivalent to the outcome of horizontal mergers in the absence of synergies, and a cartel of size K can be sustained ($\delta_K < 1$) if $K \geq \frac{3}{2} + N - \frac{1}{2}(\sqrt[2]{4N+5})$.[11] In this case, although

[10] It can be verified that $f(N) \geq \frac{3}{5}N$. Therefore, if $K < \frac{3}{5}N$, then δ_K is always decreasing in K.

[11] Note that this inequality is exactly the same in Salant et al. (1983) when firms merge: for any N, it is sufficient for a merger between K firms to be unprofitable that less than 80% of the firms merge.

cartels (and mergers) increase price they are generally not profitable because non participating firms react to the cartel (or merger) expanding their production.

5. Concluding comments

We have developed a theoretical framework to study how the number of firms in a cartel affects the possibility that its members can sustain a collusive agreement. We show that, when firms compete in quantities, collusion is easier to sustain the larger the cartel is. Also, by adding an initial stage to the usual repeated game with colluding firms, we obtain an endogenous cartel size. The sustainable cartel of the smallest size is the unique stable cartel. The intuition is that firms may have incentives to participate in the cartel only when the sustainability of collusion depends on their participation.

The framework we have worked with is, admittedly, a particular one. To analyze real-world cartels, additional research is required, and for instance a wider range of demand functions should also be considered. We believe that those are subjects for future research.

Appendix

Proof of Proposition 1. It can be verified that

$$\delta_K = \frac{(1+2c)^2(K-1)(1+2c+N)^2}{(-1+2c(K-1)+2K-N)(1+16c^3+N+8c^2(3+K+N)+k(5-2K+3N)+c(10-2K^2+8N+4K(3+N)))}.$$

It is tedious but straightforward to show that, if $K \leq N$, then $\frac{\partial \delta_K}{\partial K} < 0$. ■

Proof of Proposition 2. If $\delta < \delta_N$ for each $K \leq N$, condition [3] is not satisfied. Then, by joining the cartel firms obtain the Cournot profits, thus conditions [4] and [6] are never satisfied. We now prove that no cartel is stable when for each $K \leq N$, $\delta \geq \delta_K$ (i.e. a cartel of any size K is sustainable as a SPNE). For given values of K and N, it is a standard exercise to verify that, in our model, internal stability is satisfied (conditions [4] and [6]) if

$$\frac{(1+2c)^2(K+c)}{(4c^2+K(2+N-K)+2c(1+K+N))^2} \geq \quad [9]$$
$$\geq \frac{(1+c)(K+2c-1)^2}{(4c^2+(K-1)(3+N-K)+2c(K+N))^2}.$$

On the other hand, external stability is satisfied (conditions [5] and [7]) if

$$\frac{(1+2c)^2(K+c)}{(4c^2+K(2N-K)+2c(1+K+N))^2} \geq \qquad [10]$$
$$\geq \frac{(1+2c)^2(1+K+c)}{(4c^2+1+N+K(N-K)+2c(2+K+N))^2}$$

It is easy to see that, since $\frac{\partial \Pi^c(N,K)}{\partial K} < \frac{\partial \Pi^f(N,K)}{\partial K}$, inequality [9] is less binding as K increases. Then, we have two different cases: 1) if $K \geq 3$. It is enough to check that if $K = 3$ condition [9] it is not satisfied for $N > 2$, and thus, it is not satisfied either if $K > 3$. 2) if $K = 2$, condition [9] it is not satisfied either for $N > 2$.

Then, from Proposition 1, if $\delta_K \leq \delta < \delta_{K-1}$ only a cartel of K firms can be sustained as a SPNE of the repeated game and can thus be stable because, otherwise, by joining the cartel firms obtain the Cournot profits and conditions [4] and [6] could not be satisfied. Therefore, it is immediate to verify that conditions [4] and [6] are satisfied: $\Pi_0^c(N,K) > \Pi_0^f(N,K-1) = \frac{1}{1-\delta}\Pi(N)$. In other words, when the cartel maximizes joint profits, the profits of a cartel firm are higher than the profits of a firm in the Cournot equilibrium, which is true $\forall N$ if $K > 2$. If $K = 2$, $\delta_K \notin (0,1)$. On the other hand, conditions [5] and [7] are also satisfied since $\Pi_0^f(N,K) > \Pi_0^c(N,K+1) = \frac{1}{1-\delta}\Pi(N)$. Namely, the profits of a fringe firm are always larger than Cournot profits. ■

Proof of Proposition 3. The conditions specified in Abreu (1986) under which all cartel firms producing q constitutes a SPNE are the following

$$\Pi_p^c(N,K) + \frac{\delta_a}{1-\delta_a}\Pi^c(N,K) = 0 \qquad [11]$$

where $\Pi_p^c(N,K)$ denote the profits obtained by a cartel firm when producing q^p, and

$$\frac{1}{1-\delta}\Pi^c(N,K) \geq \Pi^d(N,K). \qquad [12]$$

We denote by δ_a the minimum discount factor needed for the condition [11] to be satisfied. Equivalently, we denote by δ_b the minimum discount factor needed for the condition [12] to be satisfied. Then, it can be verified that

$\delta_a = \frac{3(2+N+K(N-K+3))^2}{16+3K^4+6K(2+N)(3+N)+3N(4+N)-2K^3(5+3N)+3K^2(1+N(4+N))}$, and $\delta_b = \frac{(K-1)^2}{(K+2)^2}$. The minimum discount factor required for a cartel of size K maximizing their joint profits at equilibrium (δ_K) will be given by the envelope from above of δ_a and δ_b, that is $\delta_K = \max\{\delta_a, \delta_b\}$. It is tedious but straightforward to check that $\frac{\partial \delta_a}{\partial K} < 0$ and $\frac{\partial \delta_b}{\partial K} > 0$. Then, δ_a is minimized if $K = N$. We just have to see up to which value of K, $\delta_a > \delta_b$. Let $g(K) = \delta_a - \delta_b$. Then, $g(K) = 0$ if $K = \frac{13+3N+\sqrt{(9N^2+138N+249)}}{10}$. If $K < \frac{13+3N+\sqrt{(9N^2+138N+249)}}{10}$, then $g(K) > 0$, and the minimum discount factor required for a cartel of size K maximizing their joint profits at equilibrium decreases in K. ■

Proof of Proposition 4. For $K = 2$, we have already demonstrated in the proof of Proposition 2 that if $\delta \geq \delta_2$, condition [9] it is not satisfied for $N > 2$. If $\delta < \delta_2$ then, $\Pi_0^c(N, K) = \Pi_0^f(N, K-1) = \frac{1}{1-\delta}\Pi(N)$ and condition [4] is not satisfied. On the other hand, it can be easily verified that if $K = 3$ then $\delta_a > \delta_b$. Hence, from the proof of Proposition 3 we know that $\forall N$, δ_K is decreasing in K if K is relatively small and increasing in K if K is relatively large. In other words, for given values of N, δ_K is a U-shaped function with respect to K and δ. Then, for $K > 2$ if $\delta < \delta_K$ then the cartel of size K is not sustainable and $\Pi_0^c(N, K) = \frac{1}{1-\delta}\Pi(N)$. Thus, condition [4] is not satisfied and no cartel is stable. If $\delta \geq \delta_{K-1}$, then we have two different cases: (i) $\delta < \min\{\delta_2, ..., \delta_N\}$ which means that δ_K decreases in K around δ. In this case, the argument used in the last part of the proof of Proposition 2 applies and no cartel is stable. (ii) If $\delta > \min\{\delta_2, ..., \delta_N\}$, then δ_K increases in K around δ. In this case, since $\delta \geq \delta_{K-1}$ the cartel of size $K-1$ is sustainable. Therefore, from the first part of the proof of Proposition 2, the cartel of size K is not stable (condition [9] is not satisfied for $K \geq 3$). On the contrary, if $\delta_K \leq \delta < \delta_{K-1}$ the cartel of size K is stable given that it is the only sustainable cartel. ■

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Resumen

En este artículo analizamos como el tamaño de un cartel afecta a las posibilidades que existen para que un acuerdo colusivo sea sostenido. Desarrollamos un modelo con un oligopolio que produce durante infinitos periodos un bien homogéneo y dónde las empresas eligen la cantidad producida. Suponemos que un subconjunto de empresas colude mientras que el resto de empresas forman el margen competitivo. En el artículo demostramos que la colusión es más fácilmente sostenible cuando el cartel es más grande. Analizamos posteriormente las implicaciones de éste resultado en los incentivos a participar en un cartel. Obtenemos que las empresas quieren participar sólo cuando, de otro modo, la colusión no puede ser sostenida.

Palabras clave: Colusión; carteles parciales; estrategias de gatillo; castigo óptimo.

Recepción del original, mayo de 2007
Versión final, enero de 2008