INVESTIGACIONES ECONÓMICAS. vol. XXVIII (2), 2004, 233-255

OVERCOMPLIANCE WITH MINIMUM QUALITY STANDARDS

LANDER BELOQUI
Mondragon Unibertsitatea
JOSÉ M. USATEGUI
Universidad del País Vasco/EHU

This paper presents an unnoticed result that may occur when an effective minimum quality standard is imposed in a duopoly model where firms decide on quality and prices in two stages and where both covered or uncovered markets may be endogenous outcomes of the competition game. We derive situations, associated to transitions induced by the standard from uncovered market to covered market, where the two firms decide to provide a quality level higher than the minimum quality required by the standard.

Keywords: Overcompliance, quality standards, market coverage, quality competition.

(*JEL* L51, D43)

1. Introduction

Overcompliance with quality standards has been reported in several markets. An example is the study of the paper and pulp industry by McClelland and Horowitz (1999). However, there are no sound and complete theoretical explanations of this behavior. For the case of a monopolist, it has been argued that incentives to build a good reputation in a context where the product is a credence good may induce overcompliance.[1]

In this article we consider a duopoly in a situation where there are no asymmetries of information about quality between firms and consumers. We prove that there are contexts where the two firms decide

Research support from the Plan Nacional de I+D+I (project BEC2000-0301), UPV (9UPV 00035.321-13511/2001) and Basque Government (GV PI-1998-86) is gratefully acknowledged. We thank two anonymous referees for helpful comments and suggestions.

[1] See Cavaliere (2000) and Kirchhoff (2000). These works focus on environmental quality.

to provide quality levels higher than the minimum quality required by an effective standard (i.e., by a standard that forces an increase in the quality of the low quality firm). Hence, we present a theoretical model of competition where overcompliance by both firms results. To our knowledge, in the previous literature there is no explanation of overcompliance by all active firms in the market, except for the case of a monopoly.[2]

We assume that there are no costs to enforce compliance with the standard and, therefore, quality levels will never be below the standard. In this context, we prove that if the market is not covered at the unregulated equilibrium, but the solution with a covered market is feasible, then there exists a range of values of the standard that yield overcompliance.

To derive the result we set forth a model where market coverage is endogenous. It is often restrictive to consider that the market is either covered or uncovered for exogenous reasons and to assume that the level of the standard does not affect the market coverage configuration. The consideration of all market coverage configurations as possible permits to analyze transitions between these configurations and to obtain situations where there is overcompliance.[3]

When market coverage is endogenous to the analysis there are three possible market configurations for a duopoly: uncovered market, market covered with corner solution and market covered with interior solution. This classification was introduced in Shaked and Sutton (1982). The market is covered with a corner solution when the low quality firm quotes the price which is just sufficient to cover the market. The market is covered with an interior solution when it is covered in the usual sense. The transition from market not covered to market covered is not smooth as the nature of market competition changes when the market is covered because firms compete just for market shares. This lack of smoothness causes the existence of the corner solution.

[2] Arora and Gangopadhyay (1995) consider a duopoly, but show that only the high quality firm overcomplies (the low quality firm just meets the standard). The reason for overcompliance by the high quality firm is to soften price competition.

[3] It may seem natural to consider that markets are not covered. However, there are markets that are covered or almost covered. The markets of several white goods are an example. If we consider that the potential market for a white good is formed by all individuals or families that live in an apartment or a house, there are many local markets for white goods in the more developed countries that are close to full market coverage.

Most of the literature on the consequences of minimum quality standards assumes as given exogenously either a covered market configuration (as in Crampes and Hollander (1995)) or an uncovered market configuration (as in Ronnen (1991)).[4] Only Boom (1995) allows for endogenous market coverage but introduces a simplifying assumption that ensures that at equilibrium the whole market will be covered (with corner solution or with interior solution). However, it is necessary to consider the market configuration as a result of the interaction between firms and government when the standard may cause transitions between market configurations. We prove, precisely, that overcompliance occurs when the market is not covered at the unregulated equilibrium and the standard induces a transition to the equilibrium of the configuration of market covered with a corner solution.

We consider a two-stage duopoly game where firms decide first simultaneously on the production technology, with its associated quality level, and then compete simultaneously in prices. Hence, we recognize that firms can change their prices in a short period of time, whereas a change in the technology takes a longer period of time. In this setting, it is clear that firms may reduce price competition through quality differentiation. We assume that there is a maximum attainable quality level and show the possibility of overcompliance in two contexts: when there are no quality costs and when there are convex fixed quality costs. In this latter context, we obtain our overcompliance result for situations where the quality of the high quality firm is equal to the maximum attainable quality.

In our analysis, consumers have diverse "taste for quality". Each consumer prefers a product of higher quality but consumers differ on the intensity of their preference for quality. We look for the (subgame perfect) equilibrium when the standard is imposed and compare this equilibrium with the unregulated equilibrium.[5]

We include a welfare analysis in this work. If there are no quality costs, welfare increases when a standard is established. This result is a consequence of the increase in the quality of the low quality firm caused by the standard and, if the market is not covered, of the increase in

[4] Scarpa (1998) and Lutz, Lyon and Maxwell (1998) are two recent developments that consider an uncovered market. Ecchia and Lambertini (1997), instead, assume a covered market.

[5] Gabszewicz and Thisse (1979) and Shaked and Sutton (1982) settled the basis for many later analysis of quality differentiation.

market coverage caused by the higher price competition induced by the standard. When there are fixed quality costs, however, a standard may not increase welfare. The reason is that a standard increases the quality costs of the low quality firm and this increase may offset the effects pointed out for the case without quality costs. Nevertheless, we show that there are situations with convex fixed quality costs where there exist standards that increase welfare and cause overcompliance. Moreover, we prove that, with fixed quality costs or without quality costs, all consumers are better off with a standard that induces overcompliance. Hence, we may consider that our analysis on the possibility of overcompliance is performed in a context where the standard is set to increase consumers, surplus or to increase welfare (in this latter case, if there are fixed quality costs, we would study overcompliance in situations where the standard increases welfare).

The paper is organized as follows: Section 2 presents the unregulated equilibrium for the case without quality costs. Section 3 proves, for this case, how there are situations where both firms overcomply with a minimum quality standard. Section 4 shows the possibility of overcompliance when there are convex fixed quality costs. A welfare analysis is performed in Section 5. The last section briefly summarizes the results and includes some comments.

2. Equilibrium in the unregulated market

Let us assume that parameter values and technology conditions are such that there are at most two firms, 1 and 2, in the market and each firm produces one product. In this context, we consider a two-stage game where firms first choose nonnegative quality levels simultaneously and then compete simultaneously in prices.[6] Firm i produces a good of quality s_i and sells it at price p_i, $i = 1, 2$. We assume that technology only allows for a maximum quality level equal to S and that firm 2 is the top quality firm. Hence, there would exist equilibria that are symmetric to the ones presented in this paper with firm 1 as the top quality firm.

Products are sold to a population of consumers differing in their marginal valuation of quality. Consumers may purchase either a single unit of the good from one of the firms or none at all. Consumers' pref-

[6] For a discussion on contexts where it is adequate to assume simultaneous choice of quality see Aoki and Prusa (1996).

erences are described as follows: a consumer, identified by j, enjoys (indirect) utility $U(j) = js - p$ when consuming a product of quality s sold at a price p.[7] His utility is zero if he refrains from buying.[8]

The population of consumers is described by the parameter j which is uniformly distributed between v and bv, with v positive and b greater than one. The assumption of v positive is necessary to allow for the possibility of a covered market. If the lowest value of the parameter j would be 0, like in Ronnen (1991), the market would not be covered as the consumer with marginal valuation of quality equal to 0 would not buy any good with positive price. We normalize the number of consumers to one and assume $v = 1$, without loss of generality.[9]

Let us denote the demand function by $D_i(p)$ for $i = 1, 2$. We know that consumer j will be willing to buy the product of firm i, with $i = 1, 2$, if $\frac{p_i}{s_i} < j$. Moreover, the consumer that is indifferent between the product of firm 1 and the product of firm 2 has j such that $js_1 - p_1 = js_2 - p_2$, i.e., $j = \frac{p_2 - p_1}{s_2 - s_1}$. Hence, we have that, when the market is not covered, $D_1(p) + D_2(p) < 1$ and

$$D_1(p_1, p_2) = \frac{1}{b-1}\left(\frac{p_2 - p_1}{s_2 - s_1} - \frac{p_1}{s_1}\right)$$

$$D_2(p_1, p_2) = \frac{1}{b-1}\left(b - \frac{p_2 - p_1}{s_2 - s_1}\right),$$

and when the market is covered, $D_1(p) + D_2(p) = 1$ and

$$D_1(p_1, p_2) = \frac{1}{b-1}\left(\frac{p_2 - p_1}{s_2 - s_1} - 1\right)$$

[7] See Peitz (1995) for the construction of a direct utility function that has as its counterpart an indirect utility function as the one used in this paper. Peitz (1995) shows that the underlying preference relation satisfies transitivity, completeness and local nonsatiation. On the other hand, additive separability is reasonable as long as the price of the product is such that the consumer expends only a small fraction of his total budget in the product.

[8] We may interpret the model in terms of environmental quality and minimum environmental standards. In this case the parameter j that characterizes each consumer would indicate intensity of preference for environmental quality.

[9] When there are not quality costs, overcompliance occurs in the same circumstances and under the same set of parameter values for any positive value of v. With fixed quality costs, instead, the set of parameter values where there may be overcompliance depends on the value of v. Besides, if v is very small (i.e., if there are consumers that care mainly about the prices of the products and not about their qualities) fixed quality costs will have to be low to allow for the possibility of full market coverage.

$$D_2(p_1, p_2) = \frac{1}{b-1}\left(b - \frac{p_2 - p_1}{s_2 - s_1}\right).$$

We consider initially a situation where there are no quality costs. Moreover, to simplify the presentation we assume throughout that production costs are zero.

We study equilibria in pure strategies. We will proceed by backward induction to look for a subgame perfect equilibrium.

The unregulated equilibrium for the case where there are no quality costs is discussed in Wauthy (1996). From the profit functions we can first derive the equilibrium outcomes for the price subgame. The parameter regions associated to each market configuration are obtained from the decision of the consumer with the lowest valuation for quality. The market is not covered if this consumer does not buy any of the two products ($v = 1 < \frac{p_1}{s_1}$), it is covered with interior solution if he has a positive surplus when buying the low quality product ($1 > \frac{p_1}{s_1}$) and it is covered with a corner solution when his surplus from buying the low quality product is zero (the rest of situations, where $1 = \frac{p_1}{s_1}$). We have (let us use the superscripts **, ^{c} and *, respectively, to denote the equilibrium values at the market configurations of market not covered, market covered with corner solution and market covered with interior solution):

i) Market not covered whenever $b > \frac{4s_2 - s_1}{s_2 - s_1}$ (or $s_1 < s_2\frac{b-4}{b-1}$):

$$p_1^{**} = b(s_2 - s_1)\frac{s_1}{4s_2 - s_1}$$

$$p_2^{**} = b(s_2 - s_1)\frac{2s_2}{4s_2 - s_1}$$

ii) Market covered with a corner solution whenever $\frac{2s_2 + s_1}{s_2 - s_1} \leq b \leq \frac{4s_2 - s_1}{s_2 - s_1}$ (or $s_2\frac{b-4}{b-1} \leq s_1 \leq s_2\frac{b-2}{b+1}$):

$$p_1^c = s_1$$

$$p_2^c = \frac{s_1 + b(s_2 - s_1)}{2}$$

iii) Market covered with an interior solution whenever $2 < b < \frac{2s_2 + s_1}{s_2 - s_1}$ (or $s_1 > s_2\frac{b-2}{b+1}$):

$$p_1^* = \frac{b-2}{3}(s_2 - s_1)$$

$$p_2^* = \frac{2b-1}{3}(s_2 - s_1)$$

The market is preempted by firm 2 whenever $1 < b \leq 2$.[10] Therefore, to focus on situations where there are two firms in the market we consider that $b > 2$.

The revenue (and profit) functions for each market configuration can now be written as follows:

i) If the market is not covered:

$$R_1^{**} = s_1 \frac{b^2 s_2 (s_2 - s_1)}{(b-1)(4s_2 - s_1)^2}$$

$$R_2^{**} = 4s_2 \frac{b^2 s_2 (s_2 - s_1)}{(b-1)(4s_2 - s_1)^2}$$

ii) If the market is covered with a corner solution:

$$R_1^c = \frac{1}{2(b-1)(s_2 - s_1)}(bs_1(s_2 - s_1) - 2s_1 s_2 + s_1^2)$$

$$R_2^c = \frac{1}{2(b-1)(s_2 - s_1)}(bs_1(s_2 - s_1) + \frac{b^2(s_2 - s_1)^2 + s_1^2}{2})$$

iii) If the market is covered with an interior solution:

$$R_1^* = (b-2)^2 \frac{s_2 - s_1}{9(b-1)}$$

$$R_2^* = (2b-1)^2 \frac{s_2 - s_1}{9(b-1)}$$

It is not difficult to show the concavity of R_1^{**} and R_1^c with respect to s_1. Moreover, R_2^{**} is concave with respect to s_2 but R_2^c is convex with respect to s_2. However, we will show that this convexity will not pose any problems for the analysis in this work. Finally, R_1^* is linear and decreasing in s_1 and R_2^* is linear and increasing in s_2. Notice also that the profits of the high quality firm are higher than the profits of the low quality firm.

[10] When $b = 2$, the consumer with the lowest valuation of quality is indifferent between buying any of the two goods. If the market is preempted by firm 2, it will be $p_2^p = s_2 - s_1$ and $p_1^p = 0$.

From these expressions of the profit functions, we obtain the quality decisions within each market configuration:

i) If the market is not covered ($b > 8$):

$$s_2^{**} = S$$
$$s_1^{**} = \frac{4}{7}S$$

ii) If the market is covered with a corner solution ($5 \leq b \leq 10$):

$$s_2^c = S$$
$$s_1^c = \frac{b-1-\sqrt{b-1}}{b-1}S$$

iii) If the market is covered with an interior solution ($b > 2$):

$$s_2^* = S$$
$$s_1^* = \frac{b-2}{b+1}S$$

The parameter regions associated to each market configuration are derived substituting the corresponding quality equilibrium decisions in the parameter regions derived for the equilibrium outcomes of the price subgame. For instance, from $s_1 < s_2\frac{b-4}{b-1}$ and $s_1^{**} = \frac{4}{7}s_2^{**} = \frac{4}{7}S$ we obtain $b > 8$. When $b < 8$, notice that R_1^{**} always increases with s_1 and the market will end up covered ($s_1 \geq \frac{b-4}{b-1}S$). Furthermore, when $\frac{b-4}{b-1}S \leq s_1 \leq \frac{b-2}{b+1}S$, we have that R_1^c increases with s_1 if $b < 5$ and that R_1^c decreases with s_1 if $b > 10$.

Since there are neither fixed costs of increasing quality nor greater variable costs to produce a good of higher quality, firm 2 decides $s_2 = S$ to reduce price competition. Hence, it is the low quality firm who determines the equilibrium market configuration. As there are parameter regions where more than one candidate to equilibrium exists, we have to compare the profits of firm 1 under each candidate in those regions to obtain the equilibrium selected for each value of b. Let us denote by NC, CC and CI, respectively, the candidates to equilibrium corresponding to the configurations of market not covered, market covered with a corner solution and market covered with an interior solution.

As R_1^* is decreasing in s_1, CI is in the boundary of the region where the market is covered with interior solution (i.e., in the boundary between that region and the region where the market is covered with a corner solution). Thus, CI corresponds in fact to a particular situation of market covered with corner solution (notice that $p_1^* = \frac{b-2}{3}(s_2^* - s_1^*) =$

s_1^*), but it differs, in general, from CC. Therefore, CC will always be preferred by firm 1 to CI when the two solutions are defined and differ, i.e., when $5 < b \leq 10$ (if $b = 5$ both solutions coincide as $\frac{b-1-\sqrt{b-1}}{b-1} = \frac{b-2}{b+1}$). When both NC and CC are defined (i.e., when $8 < b \leq 10$), it may be shown that $R_1^{**}(s_1^{**}, S) \gtreqless R_1^c(s_1^c, S) \Leftrightarrow b \gtreqless 8.6581$. Finally, it is easy to check that the profits of firm 1 at NC are greater than the profits of this firm at CI when the two solutions are defined, i.e., when $b > 8$.

Hence, we have that at the unregulated subgame perfect equilibrium:

i) If $b \geq 8.6581$, the market will not be covered and:

$$s_2^{**} = S \qquad p_2^{**} = \frac{2bS}{4S - s_1^{**}}(S - s_1^{**})$$

$$s_1^{**} = \frac{4}{7}S \qquad p_1^{**} = \frac{bs_1^{**}}{4S - s_1^{**}}(S - s_1^{**})$$

ii) If $8.6581 \geq b \geq 5$, the market will be covered with a corner solution and:

$$s_2^c = S \qquad p_2^c = \frac{s_1^c + b(S - s_1^c)}{2}$$

$$s_1^c = \frac{b-1-\sqrt{b-1}}{b-1}S \qquad p_1^c = s_1^c$$

iii) If $5 \geq b > 2$, the market will be covered with an interior solution and:

$$s_2^* = S \qquad p_2^* = \frac{2b-1}{3}(S - s_1^*)$$

$$s_1^* = \frac{b-2}{b+1}S \qquad p_1^* = \frac{b-2}{3}(S - s_1^*)$$

To complete the derivation of the unregulated (subgame perfect) equilibrium description we must prove that there are no incentives to leapfrog the rival at equilibrium. We proceed now to consider this aspect.

Notice first that upward leapfrogging by the entrant is not feasible as $s_2 = S$. To prove that backward leapfrogging is not profitable for the high quality firm, we proceed as follows: Remember that the profits of the high quality firm are higher than the profits of the low quality firm at the equilibrium in each market configuration. Moreover, notice that the profits of the low quality firm increase with the quality selected by the high quality firm. Let us denote by s_2^b and R_2^b the quality and profits of firm 2 when this firm leapfrogs firm 1 and becomes the low quality firm, and by s_1^e the level of quality of the low quality firm in the equilibrium ($s_1^e = s_1^{**}$ when the market is not covered, $s_1^e = s_1^c$ when the market is covered with a corner solution and $s_1^e = s_1^*$ when the market is covered with an interior solution). Then we have $s_2^b < s_1^e$ and

$$R_2^b(s_2^b, s_1^e) \equiv R_1(s_2^b, s_1^e) < R_1(s_2^b, S) < R_1(s_1^e, S) < R_2(s_1^e, S)$$

Hence, there will not be backward leapfrogging.

3. Minimum quality standards and overcompliance

Consider a minimum quality standard (MQS) equal to αS, with $\alpha < 1$. Assume that αS is greater than the equilibrium value of s_1. Proposition 1 states in which situations there is overcompliance with the MQS when there are no quality costs:

PROPOSITION 1. *When $8.6581 < b < 10$ (i.e., when the market configuration without the standard is the one corresponding to uncovered market and the configuration of market covered with corner solution is feasible), there exists a value $s_k \in \left(s_1^{**}, \frac{b-4}{b-1}S\right)$ such that if $\alpha S \in (s_k, s_1^c)$ there will be overcompliance*[11].

PROOF: When the market is not covered, the profit function of firm 1 is decreasing in s_1 for values of s_1 such that $s_1^{**} < s_1 < \frac{b-4}{b-1}S$. Besides, when the market is covered with a corner solution the profit function of firm 1 is increasing in s_1 for values of s_1 such that $\frac{b-4}{b-1}S < s_1 < s_1^c$. Let us then define a quality level s_k such that $s_1^{**} < s_k < \frac{b-4}{b-1}S$ and the profits of firm 1 if it decides s_k (market not covered) are equal to its profits if it chooses s_1^c (market covered with corner solution). This s_k solves

$$\frac{b^2(S - s_k)s_k S}{(b-1)(4S - s_k)^2} = \frac{s_1^c}{2(b-1)}\left[b - \frac{2S - s_1^c}{S - s_1^c}\right]$$

From the definition of s_k, we have that when $\frac{s_k}{S} < \alpha < \frac{b-4}{b-1}$ firm 1 has lower profits deciding $s_1 = \alpha S < \frac{b-4}{b-1}S$ (uncovered market) than choosing $s_1 = s_1^c$, the equilibrium corner solution in a covered market. Moreover, since $\pi_1(s_k) = \pi_1(s_1^c)$, we have that when $\frac{b-4}{b-1} < \alpha < \frac{s_1^c}{S}$ firm 1 has lower profits deciding $s_1 = \alpha S$ than choosing $s_1 = s_1^c$. As a consequence, if $\frac{s_k}{S} < \alpha < \frac{s_1^c}{S}$ firm 1 will decide $s_1 = s_1^c$ and there

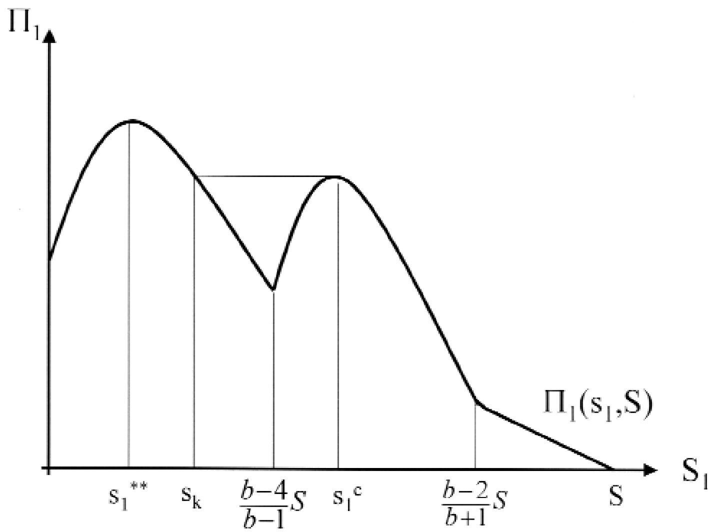
[11] Notice that $\frac{b-4}{b-1}S < \frac{b-1-\sqrt{b-1}}{b-1}S$ $(= s_1^c)$ if and only if $b < 10$.

will be overcompliance. From the analysis of leapfrogging performed in the case of the unregulated equilibrium we have that, when $s_1 = s_1^c$, backward leapfrogging (to a quality level between αS and s_1^c) is not profitable for the high quality firm. ■

An effective standard always increases the quality of the good produced by firm 1. But in the case of Proposition 1 this increase in quality is greater than the increase required by the MQS. The reason for this result is the variation in market configuration, from market not covered to market covered with a corner solution, induced by the standard. Although the standard per se may not necessarily require that the market be covered, the reaction of firms to the standard implies full market coverage.

In Figure 1 we depict the profit function of the low quality firm for the situation considered in Proposition 1 (notice that without quality costs we always have $s_2 = S$ at equilibrium). This Figure also shows that there is no overcompliance when the standard induces a transition from the configuration of market covered with corner solution to the configuration of market covered with interior solution. The reason is that firm's 1 profits are monotonically decreasing in its level of quality in the configuration of market covered with an interior solution, and hence the selection of firm 1 in this latter market configuration is always a boundary, which eliminates the possibility of overcompliance.

FIGURE 1
Profit function of the low quality firm when 8.6581<b<10



Consider, as an example, that $b = 9$. In this case, we have $s_1^{**} = 0.5715S$, $\frac{b-4}{b-1}S = 0.625S$ and $s_1^c = 0.646S$, and we obtain $s_k = 0.618S$. Hence, when $b = 9$ an standard αS such that $0.618 < \alpha < 0.646$ would induce overcompliance by the low quality firm.

4. Quality costs and overcompliance

The result of overcompliance of the MQS presented in Proposition 1 may be extended to situations where there are quality costs, i.e., costs that increase with the quality level selected. In the literature there are analyses that consider, in contexts where market coverage is given exogenously, either fixed quality costs, that do not depend on the production level, as in Ronnen (1991), Aoki and Prusa (1996) and Constantatos and Perrakis (1999), or variable quality costs, that depend on the production level, as in Crampes and Hollander (1995).

We consider that firms face only fixed quality costs. These costs may be considered as the ones required to incorporate (once and for all) the technology associated to the corresponding quality level. For instance, the fixed costs of quality may be incurred during the research and development phase of the product. The case of variable quality costs is not considered in this work. Variable quality costs affect the equilibrium of the price subgame, contrary to the case of fixed quality costs, and, therefore, call for a complete new derivation of the results. Moreover, as we need fixed quality costs to have firms committed to a certain quality level during price competition, variable quality costs, if considered, would have to go with fixed quality costs in the analysis.

It is usually assumed that the fixed quality cost function is convex and we incorporate this assumption in our analysis. When there are convex fixed quality costs, an equilibrium with two firms may not exist due to the incentives to leapfrog.[12] Hence, the functional form assumed for the convex fixed quality cost function, together with the rest of parameters, must avoid this difficulty and make possible the existence of an equilibrium with two firms.

The profit functions of the firms in each market configuration may be obtained subtracting fixed quality costs from the revenue functions presented in section 2. Let us use letter π for profits and represent the fixed quality cost function by $c(s)$, with $c'(s) > 0$ and $c''(s) > 0$. The reaction functions corresponding to the quality decisions of firms are:

[12] See the analysis in Lehman-Grube (1997) for the case of market not covered.

i) If the market is not covered:

$$\frac{\partial \pi_1^{**}}{\partial s_1} = 0 \Longrightarrow \frac{b^2 s_2^2 (4s_2 - 7s_1)}{(b-1)(4s_2 - s_1)^3} - c'(s_1) = 0 \qquad [1]$$

$$\frac{\partial \pi_2^{**}}{\partial s_2} = 0 \Longrightarrow \frac{4b^2 s_2 (4s_2^2 - 3s_1 s_2 + 2s_1^2)}{(b-1)(4s_2 - s_1)^3} - c'(s_2) = 0 \qquad [2]$$

ii) If the market is covered with a corner solution:

$$\frac{\partial \pi_1^c}{\partial s_1} = 0 \Longrightarrow \frac{1}{2} - \frac{s_2^2}{2(b-1)(s_2 - s_1)^2} - c'(s_1) = 0 \qquad [3]$$

$$\frac{\partial \pi_2^c}{\partial s_2} = 0 \Longrightarrow \frac{b^2}{4(b-1)} - \frac{s_1^2}{4(b-1)(s_2 - s_1)^2} - c'(s_2) = 0 \qquad [4]$$

iii) If the market is covered with an interior solution:

$$\frac{\partial \pi_1^*}{\partial s_1} = -\frac{(b-2)^2}{9(b-1)} - c'(s_1) < 0 \qquad [5]$$

$$\frac{\partial \pi_2^*}{\partial s_2} = 0 \Longrightarrow \frac{(2b-1)^2}{9(b-1)} - c'(s_2) = 0 \qquad [6]$$

The equilibrium for each market configuration derives from the corresponding conditions in [1] to [6] (notice that second order conditions are satisfied). Under convex fixed quality costs, the profit function of firm 1 when the market is not covered and the profit function of firm 1 when the market is covered with a corner solution are concave with respect to s_1. Moreover, π_1 decreases with s_1 when the market is covered with an interior solution. Hence, condition [5] implies that s_1 will equal the minimum value that permits to attain this market configuration.

The profit function of firm 2 when the market is not covered and the profit function of firm 2 when the market is covered with an interior solution are concave with respect to s_2. The feasibility of the configuration of market covered with corner solution requires that the convexity of the cost function compensates the convexity of the revenue function of firm 2 in such a way that the profit function of firm 2 is concave within this configuration. If the profit function of firm 2 is convex in the configuration of market covered with corner solution, it will never result this configuration at equilibrium.

To show that the result of overcompliance with the MQS presented in Proposition 1 may be extended to contexts where there are quality costs, we are going to consider situations where $s_2 = S$ at equilibrium. This would occur if S is low and/or quality costs do not increase very fast with quality. We would also have $s_2 = S$ when only the low quality firm suffers from quality costs, maybe because the high quality firm has already paid for the technology to produce with high quality and the low quality firm wants to mimic the technology of the other firm.[13] When $s_2 = S$, the value of s_1 in any market configuration may be obtained from the corresponding reaction function of firm 1.

If there are convex fixed quality costs, the values of b where each market configuration is feasible, and the values of b where each market configuration is obtained at the unregulated equilibrium, depend on the function $c(s)$. Moreover, even for simple fixed quality cost functions and considering situations where $s_2 = S$, it is not possible to obtain either s_1^{**} or s_1^c in an explicit way. Nevertheless, we can extend the result in Proposition 1 to situations where there are fixed quality costs in the following way:

PROPOSITION 2. *If $c(s)$, S and b are such that:*

i) the market configurations of market not covered and market covered with corner solution are feasible,

ii) at the unregulated equilibrium candidates of those market configurations it is $s_2 = S$ and the profits of the high quality firm are higher than the profits of the low quality firm, and

iii) without standard the market is not covered,
there exists a value $s_k \in (s_1^{**}, \frac{b-4}{b-1}S)$ such that if $\alpha S \in (s_k, s_1^c)$ there will be overcompliance.

PROOF: We can proceed as in the proof of Proposition 1. The only difference is that the quality level s_k, such that $s_1^{**} < s_k < \frac{b-4}{b-1}S$ and the profits of firm 1 if it decides s_k (market not covered) are equal to its profits if it chooses s_1^c (market covered with corner solution), solves now

$$\frac{b^2(S - s_k)s_k S}{(b-1)(4S - s_k)^2} - c(s_k) = \frac{s_1^c}{2(b-1)}\left[b - \frac{2S - s_1^c}{S - s_1^c}\right] - c(s_1^c).$$

[13] Notice that when $c(s_2) = 0$ it is $s_2 = S$ in all market configurations (we have $\frac{\partial \pi_2^{**}}{\partial s_2} > 0$ as $4(s_2)^2 > 3s_1 s_2$, $\frac{\partial \pi_2^c}{\partial s_2} > 0$ as $b^2(s_2 - s_1)^2 - (s_1)^2 = b^2((s_2)^2 - 2s_1 s_2) + (b^2 - 1)(s_1)^2 > 0$ and also $\frac{\partial \pi_2^*}{\partial s_2} > 0$).

Notice that condition ii) permits to prove, as in section 2, that backward leapfrogging is not profitable for the high quality firm. ∎

We illustrate the result in Proposition 2 with a representative example: Example 1. We use numerical methods (Maple 4) in the computations.

EXAMPLE 1

Consider that $c(s) = \frac{s^2}{7}$ and $S = 1$. For this case, it may be shown that there may exist a market not covered if $b > 6.594$, a market covered with a corner solution if $2.664 < b < 7.304$ and a market covered with an interior solution if $b > 2$. To obtain these parameter regions we require that s_1^{**}, s_1^c and s_1^* belong to the (corresponding) intervals for each market configuration in the price subgame when $s_2 = S = 1$, and check that the quality level decided by firm 2 is equal to S in each of these regions, given the values obtained for s_1^{**}, s_1^c and s_1^*.[14] It is not difficult to verify that firms profits, which increase with b, are positive in those parameter regions and that, at the equilibrium candidate of each market configuration, the profits of the high quality firm are higher than the profits of the low quality firm.

Reasoning as in the situation without quality costs, it may be shown that the configuration of market covered with interior solution will never result at the unregulated equilibrium if any other configuration (market covered with a corner solution or market not covered) is feasible. When the solutions of the configurations of market not covered and market covered with a corner solution are defined (i.e., when $6.594 < b < 7.304$), it is obtained that $\pi_1^{**}(s_1^{**}, S) \gtreqless \pi_1^c(s_1^c, S) \Leftrightarrow b \gtreqless 6.874$. Hence, at the unregulated equilibrium the market will not be covered if $b \geq 6.874$, it will be covered with a corner solution if $2.664 < b \leq 6.874$ and it will be covered with an interior solution if $2 < b \leq 2.664$. Proceeding as in section 2, it may be proved that there are not incentives to leapfrog the rival at the unregulated equilibrium in this case.

From these results it is clear that if $6.874 < b < 7.304$, conditions i) to iii) of Proposition 2 are fulfilled and, hence, there exists a value $s_k \in (s_1^{**}, \frac{b-4}{b-1}S)$ such that if $\alpha S \in (s_k, s_1^c)$ there will be overcompliance.

Consider, for instance, that $b = 7$. In this case, we have $s_1^{**} = 0.468$, $\frac{b-4}{b-1}S = 0.5$ and $s_1^c = 0.514$, and we obtain $s_k = 0.496$. Hence, when

[14] All s_2^{**}, s_2^c and s_2^* obtained are higher than S.

$b = 7$ an standard αS such that $0.496 < \alpha < 0.514$ would induce overcompliance by the low quality firm. ■

The case where $s_2 < S$ is more complex to analyze. When $s_2 < S$, the equilibrium market configuration is determined by the quality decisions of both firms and transitions between each pair of market configurations are difficult to derive. Moreover, when a minimum quality standard is introduced, we would have to consider the possibility that any of the two firms may be interested in inducing a shift to other market configuration and, therefore, specify the dynamics of this change of market configuration (both firms coordinate to move to the new market configuration or one firm leads the shift and the other follows).

5. Welfare analysis

We show first that, if there are no quality costs, an effective standard always increases social welfare (measured as the sum of consumer surplus and producer surplus). This result is a consequence of the increase in the quality of the low quality firm caused by the standard and, if the market is not covered, of the increase in market coverage caused by the higher price competition induced by the standard. Let us denote social welfare by W. We may write in this case:

$$W = \int_{\frac{p_1}{s_1}}^{\frac{p_2-p_1}{s_2-s_1}} \theta s_1 d\theta + \int_{\frac{p_2-p_1}{s_2-s_1}}^{b} \theta s_2 d\theta$$

When $s_1 = \alpha S$ and $s_2 = S$ (the case of overcompliance with the standard will be considered bellow), we have:

i) If the market is not covered:

$$W = \frac{b^2(12-\alpha-\alpha^2)S}{2(4-\alpha)^2}$$

$$\frac{dW}{d\alpha} = \frac{b^2(20-17\alpha)S}{2(4-\alpha)^3} > 0$$

ii) If the market is covered with a corner solution:

$$W = \frac{(4(1-\alpha)(b^2-\alpha)-(b(1-\alpha)-\alpha)^2)S}{8(1-\alpha)}$$

$$\frac{dW}{d\alpha} = \frac{((b^2+2b-3)(1-\alpha)^2-1)S}{8(1-\alpha)^2}$$

and this derivative is positive for $b > 2$, noting that $\alpha \leq \frac{b-2}{b+1}$.

iii) If the market is covered with an interior solution:

$W = \frac{(\alpha(b^2+2b-8)+(8b^2-2b-1))S}{18}$

$\frac{dW}{d\alpha} = \frac{(b^2+2b-8)S}{18} > 0$

as $b > 2$.

The effect of overcompliance on welfare would be analogous to the effect of an increase in α in the previous expressions. Hence, an effective standard increases social welfare.

When there are fixed quality costs, a standard may not increase welfare. The reason is that the quality costs of the low quality firm increase with the standard and this increase may offset the effects pointed out for the case without quality costs. However, social welfare increases with the standard in the situations, considered in the previous section, where there is overcompliance in Example 1.

To study the effect of the standard on welfare in the case of Example 1, notice that, with the quality cost function considered, social welfare may be written as:

$$W = \int_{\frac{p_1}{s_1}}^{\frac{p_2-p_1}{s_2-s_1}} \theta s_1 d\theta + \int_{\frac{p_2-p_1}{s_2-s_1}}^{b} \theta s_2 d\theta - \frac{(s_1)^2}{7} - \frac{(s_2)^2}{7}$$

We only have to study if welfare increases with the standard in the configurations of market not covered and market covered with a corner solution, as these are the market configurations that are relevant for the result on overcompliance.

Considering that $s_1 = \alpha S$ and $s_2 = S$ (the case of overcompliance with the standard will be discussed later), we have:

i) If the market is not covered:

$W = \frac{b^2(12-\alpha-\alpha^2)S}{2(4-\alpha)^2} - \frac{(\alpha S)^2}{7} - \frac{S^2}{7}$

$\frac{dW}{d\alpha} = \frac{b^2(20-17\alpha)S}{2(4-\alpha)^3} - \frac{2\alpha S^2}{7}$

and this derivative is positive for $S = 1$ and $b \geq 6.874$.

ii) If the market is covered with a corner solution:

$W = \frac{(4(1-\alpha)(b^2-\alpha)-(b(1-\alpha)-\alpha)^2)S}{8(1-\alpha)} - \frac{(\alpha S)^2}{7} - \frac{S^2}{7}$

$\frac{dW}{d\alpha} = \frac{((b^2+2b-3)(1-\alpha)^2-1)S}{8(1-\alpha)^2} - \frac{2\alpha S^2}{7}$

and this derivative is positive for $b > 2.664$, noting that $\alpha \leq \frac{b-2}{b+1}$.

As in the case without quality costs, the effect of overcompliance on welfare would be analogous to the effect of an increase in α in the previous expressions.[15]

Finally, we can prove the following result that is valid for situations with fixed quality costs and also for situations without quality costs:

PROPOSITION 3. *All consumers are better off with a standard that induces overcompliance*

PROOF: We proceed through the following steps:

i) It is clear that consumers who were not participating in the unregulated market and participate in the market once the standard is set are better off.

ii) If there are consumers who purchased the product of firm 1 in the unregulated market and continue purchasing the low quality product once the standard is set, these consumers are also better off. The consumer who was indifferent in the unregulated market between purchasing the product of firm 1 and not participating in the market ($j = \frac{p_1^{**}}{s_1^{**}}$) is better off as $\frac{p_1^{**}}{s_1^{**}} > v = 1$ implies

$$\frac{p_1^{**}}{s_1^{**}}(s_1^c - s_1^{**}) - (p_1^c - p_1^{**}) = \frac{p_1^{**}}{s_1^{**}}s_1^c - s_1^c = s_1^c(\frac{p_1^{**}}{s_1^{**}} - 1) > 0$$

For any other consumer who continues buying the low quality product once the standard is set we have $j > \frac{p_1^{**}}{s_1^{**}}$ and, hence, $j(s_1^c - s_1^{**}) - (p_1^c - p_1^{**}) > 0$.

iii) Those consumers who purchased the product of firm 2 in the unregulated market and continue purchasing the high quality product once the standard is set are also better off. The reason is that, when the standard is set, the quality they obtain (S) does not change but the price they pay diminishes. This decrease in price is due to the increase in the quality of the low quality product induced by the standard and to the following results that may be obtained from the equilibrium outcomes for the price subgame, noting that $s_2 = S$:

$$\frac{dp_2^{**}}{ds_1} = 2bS(\frac{-4S + s_1 + S - s_1}{(4S - s_1)^2}) = -\frac{6bS^2}{(4S - s_1)^2} < 0$$

[15] When there are fixed quality costs, welfare may decrease with the standard within the configuration of market covered with an interior solution. However, we know that if this configuration results at the unregulated equilibrium, overcompliance cannot occur.

$$p_2^{**}(s_1 = \frac{b-4}{b-1}S) = b(S - \frac{b-4}{b-1}S)\frac{2S}{4S - \frac{b-4}{b-1}S} = 2S$$

$$p_2^c(s_1 = \frac{b-4}{b-1}S) = \frac{\frac{b-4}{b-1}S + b(S - \frac{b-4}{b-1}S)}{2} = 2S$$

and

$$\frac{dp_2^c}{ds_1} = \frac{1-b}{2} < 0$$

iv) Finally, as we will show below, some consumers who purchased the product of firm 1 in the unregulated market decide to buy the high quality product once the standard is set. These consumers are also better off because their surplus would have increased if they would have continued purchasing the low quality product (see ii)). If they decide to buy the high quality product once the standard is set, their surplus must be higher purchasing this product than buying the low quality product. To prove that there are consumers who purchased the product of firm 1 in the unregulated market and buy the high quality product once the standard is set, we show that the j of the consumer indifferent between purchasing the product of firm 1 and the product of firm 2 diminishes with the standard. As s_1 increases with the standard we obtain, from the equilibrium outcomes for the price subgame (with $s_2 = S$):

$$\frac{p_2^{**} - p_1^{**}}{S - s_1} = \frac{b(2S - s_1)}{4S - s_1}$$

$$\frac{d\frac{p_2^{**} - p_1^{**}}{S - s_1}}{ds_1} = -\frac{2bS}{(4S - s_1)^2} < 0$$

$$\frac{p_2^c - p_1^c}{S - s_1} = \frac{b(S - s_1) - s_1}{2(S - s_1)}$$

$$\frac{p_2^{**} - p_1^{**}}{S - s_1}(s_1 = \frac{b-4}{b-1}S) = \frac{b(2S - \frac{b-4}{b-1}S)}{4S - \frac{b-4}{b-1}S} = \frac{b+2}{3}$$

$$\frac{p_2^c - p_1^c}{S - s_1}(s_1 = \frac{b-4}{b-1}S) = \frac{\frac{3bS}{b-1} - \frac{b-4}{b-1}S}{\frac{6S}{b-1}} = \frac{b+2}{3}$$

and

$$\frac{d\frac{p_2^c - p_1^c}{S - s_1}}{ds_1} = \frac{1}{2}(\frac{(-b-1)(S - s_1) + b(S - s_1) - s_1}{(S - s_1)^2}) = \frac{-S}{2(S - s_1)^2} < 0$$

This proof is not affected by the existence of fixed quality costs as these costs do not modify the equilibrium outcomes for the price subgame. ■

6. Conclusion

In this work we have shown, for a duopoly model and a context where market coverage is endogenous, how overcompliance with a minimum quality standard may occur. We have found situations, associated to transitions induced by the standard from market not covered to market covered with a corner solution, where the two firms decide to provide a level of quality higher than the minimum quality standard. In some of these situations, the market would have remained not covered if the low quality firm would have chosen a quality level just equal to the standard, but the reaction of firms to the standard implies overcompliance and full market coverage.

To derive the results our study has not been limited to contexts where the market is always not covered or to situations where the market is always covered with an interior solution. We take into account all market configurations simultaneously. This is necessary because our overcompliance results follows from transitions between market configurations induced by the standard.

We have proved the possibility of overcompliance in two contexts: when there are no quality costs and when there are convex fixed quality costs. When there are no quality costs, social welfare increases when a standard is established. When there are convex fixed quality costs, we have shown that there are situations where there exist standards that cause overcompliance and increase welfare. In both contexts we have obtained that all consumers are better off with a standard that induces overcompliance.

In our work there are values of the parameters such that the "natural oligopoly" implies more than two firms.[16] However, the analysis of the equilibrium when market coverage is endogenous becomes complex if there are more than two active firms in the market. For an analysis

[16] See Shaked and Sutton (1983). We also know, from Shaked and Sutton (1982), that the maximum number of firms that may have a positive market share at equilibrium increases with the heterogeneity of consumers tastes (or rents) and their result would also apply to our work.

with three firms in a context where the market coverage is exogenously given (uncovered market) see Scarpa (1998).

When firms face fixed quality costs, we have considered in this work that both firms stay in the market after the standard is established. However, firms profits diminish, in general, with the standard (the only exception, referred to the profits of the low quality firm when the market is not covered and the standard is very mild, was pointed out in Ronnen (1991)). If the profits of a firm become negative when the standard is set, that firm would leave the market. Nevertheless, we have shown that there are situations with fixed quality costs in the duopoly model where standards cause overcompliance and firms, profits remain positive.

References

Aoki, R. and T.J. Prusa (1996): "Sequential versus simultaneous choice with endogenous quality", *International Journal of Industrial Organization* 15, pp. 103-121.

Arora S. and S. Gangopadhyay (1995): "Toward a theoretical model of voluntary overcompliance", *Journal of Economic Behavior and Organization* 28, pp. 289-309.

Boom, A. (1995): "Asymmetric international minimum quality standards and vertical differentiation", *The Journal of Industrial Economics* 63, pp. 101-119.

Cavaliere, A. (2000): "Overcompliance and voluntary agreements", *Environmental and Resource Economics* 17, pp. 195-202.

Constantatos, C. and S. Perrakis (1999): "Free entry may reduce total willigness-to-pay", *Economics Letters* 62, pp. 105-112.

Crampes, C. and A. Hollander (1995): "Duopoly and quality standards", *European Economic Review* 39, pp. 71-82.

Ecchia, G. and L. Lambertini (1997): "Minimum quality standards and collusion", *The Journal of Industrial Economics* 65, pp. 101-113.

Gabszewicz, J. and J.-F. Thisse (1979): "Price competition, quality and income disparities", *Journal of Economic Theory* 20, pp. 340-359.

Kirchhoff, S. (2000): "Green business and blue angels: A model of voluntary overcompliance with asymmetric information", *Environmental and Resource Economics* 17, pp. 403-420.

Lehmann-Grube, U. (1997): "Strategic choice of quality when quality is costly: the persistence of the high quality advantage", *RAND Journal of Economics* 28, pp. 372-384.

Lutz, S., T. P. Lyon y J. W. Maxwell (1998): "Strategic quality choice with minimum quality standards", CEPR Discussion Paper 1793.

McClelland, J.D. and J.K. Horowitz (1999): "The cost of water pollution regulation in the pulp and paper industry", *Land Economics* 75, pp. 220-232.

Peitz, M. (1995): "Utility maximization in models of discrete choice", *Economics Letters* 49, pp. 91-94.

Ronnen, U. (1991): "Minimum quality standards, fixed costs and competition", *RAND Journal of Economics* 22, pp. 490-504.

Scarpa, C. (1998): "Minimum quality standards with more than two firms", *International Journal of Industrial Organization* 16, pp. 665-676.

Shaked A. and J. Sutton (1982): "Relaxing price competition through product differentiation", *Review of Economic Studies* 69, pp. 3-13.

Shaked A. and J. Sutton (1983): "Natural oligopolies", *Econometrica* 51, pp. 1469-1483.

Wauthy, X. (1996): "Quality choice in models of vertical differentiation", *The Journal of Industrial Economics* 64, pp. 345-353.

Resumen

En este artículo se obtiene un resultado que se produce cuando se establece un estándar efectivo de calidad mínima en un duopolio en el que las empresas compiten en calidades y precios en dos etapas y en el que la cobertura del mercado se determina endógenamente como consecuencia del juego competitivo. Cuando el estándar induce una transición desde mercado no cubierto a mercado cubierto, las dos empresas deciden niveles de calidad mayores que el nivel de calidad mínimo exigido por el estándar.

Palabras clave: estándar de calidad, cobertura del mercado, competencia en calidad, cumplimiento con el estándar.

Recepción del original, enero de 2001
Versión final, septiembre de 2003