

## EQUILIBRIUM SEARCH MODELS: THE ROLE OF THE ASSUMPTIONS

J. IGNACIO GARCÍA-PÉREZ  
*Universitat Pompeu Fabra*

*This paper presents a survey of the most recent literature about Equilibrium Search Models with wage posting. Starting with the basic Burdett and Mortensen (1998) model, I describe the main consequences of departing from its two main assumptions: random matching and a linear production function. I show how the specific modeling of either the production or the matching technology can affect the results regarding the distribution function of offered wages. The main empirical results from the structural estimations of these models are also introduced and discussed.*

*Keywords: Equilibrium Search, random and balanced matching, returns to scale.*

(JEL: D83, J64)

### 1. Introduction

It is quite accepted nowadays that the traditional description of labor markets using aggregate demand and supply functions lacks realism. Labor markets are characterized by large flows of workers moving from unemployment to employment and viceversa, and moving from one job to another<sup>1</sup>. Both employers and workers are incompletely informed about other agents' strategies and about trading prices. Moreover, it takes effort and time to locate a suitable partner and to complete a transaction which, in this context, will be called a *match* between a worker and a firm.

This paper is the result of my visit to CREST, Paris and Tinbergen Institute, Amsterdam in the Winter of 1999. I am very grateful to Jean-Marc Robin and Gerard Van den Berg, to my advisor Samuel Bentolila and to two referees for all their helpful comments. I have also benefited from discussions with Arantza Gorostiaga (CEMFI), Sebastien Roux (CREST) and other professors and students in both institutes. Of course, all remaining errors are mine.

<sup>1</sup>See, for example, Davis and Haltiwanger (1990) or Burda and Wyplosz (1994).

During the past two decades, labor market research based on the principle of information uncertainty has made considerable progress in explaining the behavior of workers looking for a new job. Job search theory has proved to be a flexible tool, in both theoretical and empirical work, for improving our understanding of some important observed facts like, for example, the duration of unemployment for different types of workers or decreasing probabilities of finding a new job as the unemployment spell lengthens<sup>2</sup>. Some important issues cannot be analyzed, however, using partial job search models, i.e., those considering only one side of the market. Some examples of this are wage determination, firms' behavior and its interaction with that of workers or the effects of policies that directly affect wages.

The role of employers has been incorporated recently in this field, with the development of the so-called *Equilibrium Search* literature. In it, supply, demand, and wage determination in the labor market are modeled jointly. There are two quite different, though related, branches in this literature. The first one deals with explaining worker and job flows, and the levels of unemployment within the rational forward-looking agent paradigm. It is usually called the *Equilibrium Unemployment Approach* and it is basically developed in Pissarides (1990) and Mortensen and Pissarides (1994). It uses and describes a fundamental relationship between the number of unemployed workers and of vacant jobs called the *matching function*. The second branch, which is known as the *Wage Posting Approach* and is the basis for this article, aims at generating wage dispersion as an equilibrium outcome in markets with frictions. It is assumed that wages are set and posted by employers, and that workers search for the highest wage. Here, search frictions are based on imperfect information. Hence, they could be regarded simply as the time required for workers to gather information about wage offers<sup>3</sup>.

The wage posting approach was motivated by a purely theoretical question. After adapting optimal stopping theory to the price search problem, economists began to wonder whether it would be possible to derive the distribution of wages that motivate wage search as a market equilibrium outcome. In particular, some researchers dealt with the

<sup>2</sup>See surveys of this literature in Mortensen (1986), Devine and Kiefer (1991) and Wolpin (1995).

<sup>3</sup>A model without search frictions would be one where the information about wage offers arrives to the worker instantaneously.

question of how to generate wage dispersion when all agents in the market are homogeneous.

Diamond (1971) was the first to solve a fully consistent equilibrium price posting game under imperfect information about offers. He found that only the monopoly (monopsony) price is offered in equilibrium if price setters are the sellers (buyers) even when the number of competitors is large. Hence, the distribution function of offered wages is not continuous but degenerate at that wage. Applied to job search, Diamond's result implies that the only offered wage is the reservation wage of unemployed workers. But if this is the case, then the offered wage will be just equal to unemployment income minus out-of-pocket costs of search. Hence, if costs of search are positive, the value of search will be lower than the value of non-participation<sup>4</sup>. Therefore, the result is that there exists no equilibrium where workers want to participate. This result of finding a degenerate wage offer equilibrium distribution is known as the *Diamond paradox*.

Different attempts have been developed in the literature to overcome this unsatisfactory result. Albrecht and Axell (1984) show that it is a consequence of the assumption that all workers have identical search costs and equal opportunity costs of employment. If this is not the case, it can be proved that there exists an equilibrium where workers want to participate, firms offer each worker exactly her reservation wage, and the distribution of offered wages is not degenerate.

But the actual solution to the *Diamond paradox* was developed in Burdett (1990), Mortensen (1990) and Burdett and Mortensen (1989, 1998). The basic idea in these papers is that employed workers can also search while holding a job, that is, there exists *on-the-job search*. We will see in the following section how these models can generate a continuous distribution function of wages offered in equilibrium. These models also bring other forceful insights as to why large firms pay more than small firms, why wages increase with tenure, and why senior workers are less mobile than junior ones. This is the reason why they have received more attention in the literature. Moreover, they represent the main theoretical framework that is estimated structurally within this branch of the literature, because it has very clear theoretical predic-

<sup>4</sup>Since participation in the labor market while not employed requires search activity and because any out-of-pocket cost of search is paid by the worker and avoided when not searching, participation requires any acceptable offer to be larger than unemployment income.

tions with respect to observable variables (wages and unemployment duration).

However, these models suffer from a well-known major empirical limitation: they imply an upward sloping distribution of wages, whereas what is typically observed is a non-monotonic and unimodal function with a long right tail. Recent papers by Bowlus, Kiefer and Neuman (1995,1998), Robin and Roux (1998), and Bontemps, Robin and Van den Berg (1999, 2000) have shown that introducing labor productivity differentials across firms delivers a wage distribution with the 'right' shape.

In this paper I will be particularly concerned with exploring the role of the underlying assumptions in the Burdett and Mortensen (1998) model (hereafter the BM model). There have been some departures from this model which have led to different results. Two basic assumptions in this model are, firstly, that the probability of matching between a worker and a firm is totally random and, secondly, that the firm's production technology is linear in labor. The first assumption refers to matching, that is, to the meeting technology or the manner in which workers contact firms, and it implies that every worker has the same probability of contacting any firm. However, this seems not to be the case in the real world, where workers have some information about their possibilities of matching with different types of firms and where firms, in fact, do not play a passive role when waiting to fill vacant jobs. Thus, we would like to investigate how the BM model's results would change when we replace the so-called *random matching* assumption by a *balanced matching* one, where the firm can influence the probability of matching. The second key assumption of the BM model implies that there is no optimal level of employment and, as a result, firms want to hire as many workers as they can at the optimal wage. We will see what happens when we move to a more realistic production function not only with labor but also with capital, the level of which is chosen by firms. In this context, there will be decreasing returns to scale to labor and, in consequence, there will be an optimal level of employment for each firm.

The present survey can be considered as complementary to the recent one in Mortensen and Pissarides (1999). In contrast to the more general approach of these authors' survey, where the two branches of the Equilibrium Search literature are fully described, here the aim is to analyze the wage posting approach much more in depth. The main ob-

jective is to discuss the consequences of changing the two assumptions referred to before. In particular, I will present a general way of looking at the papers dealing with these two assumptions. More specifically, I will present a generalization of the matching probability helping us to relate all these papers and to understand better the foundations and results of Equilibrium Search models with wage posting.

Finally, this article points to a future extension for those models, namely dealing with worker heterogeneity in terms of their productivity. Workers' heterogeneity has usually been considered with regard to their valuation of time or their cost of search. However, productivity differences across workers also matter. If firms are hiring workers of different abilities in the same, non-segmented labor market, their hiring strategies should be different. The literature considering differences between skilled and unskilled workers is increasing (see, for example, Sneessens and Shadman-Mehta, 1995, Gregg and Manning, 1997, or Johnson and Sta/ord, 1997) and this issue also has to be addressed in the wage posting approach.

The structure of the paper is as follows. After presenting in Section 2 the basic BM model, where I introduce the notation used throughout the paper, two basic assumptions of this model are analyzed in Sections 3 and 4, respectively. Then, Section 5 briefly reviews the state of the art in the structural estimation of this type of models<sup>5</sup> and Section 6 concludes.

## 2. The Burdett-Mortensen model

Among those models trying to answer to the so-called *Diamond paradox*, one of the most successful is the model of Burdett and Mortensen (1998)<sup>6</sup>. It presents a wage posting game under imperfect information and search frictions where the assumption used to overcome the Diamond's result is that workers are allowed not only to search when they are unemployed but also on the job. Under this assumption, BM can prove that the steady-state equilibrium is unique and is characterized by a nondegenerate distribution of wage offers even when all workers and jobs are respectively homogeneous. This is the main reason why the wage posting approach is based on this model. Moreover, almost

<sup>5</sup>See Van den Berg (1999) for a more complete description of these techniques.

<sup>6</sup>The first version dates back to 1989.

all empirical papers estimating Equilibrium Search models use it as the basic theoretical counterpart (see Section 5).

In this model there exists a continuum of workers,  $a$ , and of employers,  $b$ , all of them respectively homogeneous. The worker can be either unemployed or employed and she can look for jobs in both states. Moreover, the arrival rates of job offers in the two states are defined as the parameter of a Poisson process for each:  $\lambda_0$  when unemployed and  $\lambda_1$  when employed. These offers come from the equilibrium distribution function of offered wages,  $Z(w)$ , which has density  $z(w)$ .

Every job-worker match can be destroyed at an exogenous rate  $\delta$ , the discount factor is  $\beta$  and an unemployed worker receives utility flow  $v$  per period.

In the worker's decision problem, as in any dynamic programming problem, we can write the expected discounted value for the two possible states and it can be proved that there exists a reservation wage,  $f$  such that an unemployed worker is indifferent between accepting this wage and not accepting it. The reservation wage has the following expression (see Mortensen and Neuman, 1988)

$$f = v + (\lambda_0 + \lambda_1) \int_f^{\infty} \frac{1 - Z(w)}{1 + (\lambda_1 [1 - Z(w)])} z(w) dw \quad [1]$$

where  $\lambda_0 = \lambda_0^0$  and  $\lambda_1 = \lambda_1^0$  and it is assumed that  $\beta < 1$ . As usual in job search models, the reservation wage is the sum of two terms: the value of unemployment time plus the expected gain from waiting for a better offer to arrive.

Given  $f$ , the flows of workers into and out of unemployment must be equal in the steady-state. Therefore, we will have that:

$$\lambda_0 [1 - Z(f)] i = \delta (a - i)$$

where  $i$  is the number of unemployed workers. As a consequence, the steady-state will be given by:

<sup>7</sup>As stated in BM, this expression comes from assuming the special case where  $\beta$  is small relative to the offer arrival rate when unemployed,  $\lambda_0$ . Of course (see BM), a more general expression for  $f$  can be obtained for other values of  $\beta$ . At  $\beta = 0$ , there are other optimizing strategies but they are of less interest for this problem.

$$i = \frac{a}{1 + \alpha [1 - Z(f)]} N \tag{2}$$

The steady-state flows of workers moving into and out of firms paying wages not greater than  $w$  will be equal in the steady-state as well, and so,

$$\alpha \int_0^w [Z(w) - Z(f)] dG(w) i = (\alpha + \beta) \int_0^w [1 - Z(w)] [G(w) - \alpha i]$$

where  $G(w)$  is the steady-state distribution of earned wages. Here, substituting for  $i$ , we can show that  $G(w)$  verifies:

$$G(w) = \frac{[Z(w) - Z(f)] \alpha [1 - Z(f)]}{1 + \beta [1 - Z(w)]} \frac{L - \alpha N}{\alpha} \tag{3}$$

Finally, we can obtain the steady-state number of workers employed in a firm offering a wage  $w$ ,  $z(w)$ , where the wages offered by other firms,  $Z(\cdot)$  and the workers' reservation wage,  $f$  are taken into account. That level is given by the steady-state number of workers earning a wage in the interval  $[w - \epsilon, w]$  over the measure of firms offering a wage in the same interval, when  $\epsilon \rightarrow 0$ . That is,

$$z(w) = \lim_{\epsilon \rightarrow 0} \left[ \frac{[G(w) - G(w - \epsilon)] (\alpha + \beta)}{[Z(w) - Z(w - \epsilon)] \alpha} \right] = \frac{G(w) (\alpha + \beta)}{Z(w) \alpha} \tag{4}$$

if  $w > f$  and  $z(w) = 0$  if  $w \leq f$ . Here,  $Z(w) = Z(w - \epsilon) + \alpha z(w)$  where  $\alpha z(w)$  is the fraction or mass of firms offering  $w$  and  $Z(w - \epsilon)$  denotes  $\lim_{\epsilon \rightarrow 0} Z(w - \epsilon)$ .

Another way of obtaining  $z(w)$  is to look at the flows into and out of those particular firms which offer a wage equal to  $w$ . The inflow is given by all unemployed workers who obtain offers higher than their reservation wage and by all employed workers earning less than  $w$  who receive an offer  $w$ . The outflow from firms paying  $w$  are those workers who are exogenously separated from the job and those who receive an offer higher than  $w$ . These two flows must be equal in steady state, thus:

$$\begin{aligned} & \int_0^{\infty} \bar{Z}(w) \lambda_1 [ (w^D)(a - i) ] z(w) x \tilde{S} = \\ & (\lambda_1 + \lambda_2 \bar{Z}(w)), (w \mid f \mathcal{Q}) b z(w) x \tilde{S} \end{aligned} \quad [5]$$

where  $\bar{Z}(w) = 1 - Z(w)$  and  $b z(w) x \tilde{S}$  is the measure of firms offering a wage  $w$  in an instant  $x \tilde{S}$

We shall call equation [5] the *steady-state equality-of-flows condition*, which will be extensively used in what follows. Note that it implicitly assumes that the probability of a match between a firm and a worker is equal for any firm in the market. That is, the probability of sampling a firm is just  $1/Q$  and, therefore, the probability of matching with a firm offering  $w$  is just  $z(w)$ . This assumption is labeled in the literature as *random matching* and it will be further analyzed in the following section.

With respect to firms' behavior, this model makes a very important assumption: there are constant returns to scale in the production function, which only depends on the number of workers. Hence, the firm's steady-state profit, given the offered wage  $w$  can be written as  $(\lambda_1 + \lambda_2 \bar{Z}(w))(w \mid f \mathcal{Q})$  where  $\lambda_1$  is the flow of revenue generated per employed worker. The strategy of the firm will be to post the wage which maximizes its steady-state profit flow.

We can now define the notion of *steady-state equilibrium* of this search and wage-posting game. It is a triple  $(f \mathcal{Q}(\cdot), \theta)$  such that:

- (i)  $f$  is the common reservation wage of unemployed workers.
- (ii)  $\theta = \max(\lambda_1 + \lambda_2 \bar{Z}(w), (w \mid f \mathcal{Q})N$
- (iii)  $Z(\cdot)$  is such that  $(\lambda_1 + \lambda_2 \bar{Z}(w), (w \mid f \mathcal{Q})) = \theta - L'$  on the support of  $Z(\cdot)$  and  $(\lambda_1 + \lambda_2 \bar{Z}(w), (w \mid f \mathcal{Q})) < \theta$  otherwise.

The main result of this model is the existence of a unique equilibrium solution if both  $E - R - \lambda_1 R - v > 0$  that is, the workers' productivity is greater than the common opportunity cost of employment, and  $E - R - (\lambda_1 + \lambda_2 \bar{Z}(w)) = 0$  which implies that offers arrive both to unemployed and employed workers.

The first characteristic of this equilibrium is that no employer will offer a wage lower than the reservation wage of unemployed workers,  $f$ . However, its main feature is that non-continuous wage offer distributions

are ruled out as equilibrium ones. This fact comes from the discontinuity of  $(w^* | f(\mathcal{Z}))$  at mass points of  $Z(\cdot)$ .<sup>8</sup> If there were a mass point  $\hat{w}$  in  $Z(\cdot)$  any employer offering a wage slightly greater than  $\hat{w}$  would have a significantly larger steady-state labor force and only a slightly smaller profit per worker than a firm offering  $\hat{w}$ . Hence, any wage just above  $\hat{w}$  would yield a greater profit and, therefore,  $\hat{w}$  cannot be an equilibrium, which precludes the result in Diamond (1971).

Finally, let us note that this model's equilibrium includes both the competitive Bertrand solution,  $w^* = \dagger$ , and Diamond's (1971) monopsony solution,  $w^* = f$ , as limiting cases. In the first case,  $\beta_1$  tends to infinity, that is, all frictions vanish. In the second case,  $\beta_1$  tends to zero, that is, employed workers cannot look for better paid jobs and, consequently, the unique equilibrium wage is  $f$ .

This basic model is extended in the same article to allow for both worker and firm heterogeneity. They will differ, respectively, in their value of leisure and in their productivity. Essentially the same results are also obtained in these two cases and therefore I will not present them any further.

To conclude, this model provides very rich insights regarding some observed facts in labor markets like, for example, that offered wages generally exceed reservation wages. Moreover, it justifies why large firms pay more than small ones, and why observationally homogeneous firms offer different wages in equilibrium. This is the reason why it received considerable theoretical and empirical attention in the past decade.

However, two characteristics of the model suffer from some lack of realism, a feature which has motivated more detailed studies about them. They have to do with two maintained assumptions: firstly, the matching technology is totally random, in the sense that the probability of meeting with a given worker is equal for every firm. And secondly, the production technology is linear, that is, the value of the marginal product of a worker is independent of the number of workers at the firm and, therefore, there is no optimal workforce for the firm. It wants to hire as many workers as it can at any given wage.

<sup>8</sup>This discontinuity can be obtained by deriving  $(w^* | f(\mathcal{Z}))$  from (5) as a function of  $\beta_0, \beta_1$  and  $Z(\cdot)$ .

In the following sections I will discuss those two assumptions in depth, summarizing the different approaches followed in the literature in relaxing them.

### 3. The matching technology: How do workers really meet firms?

In the steady-state equality-of-flows condition, equation [5], a basic element is the probability with which workers match or meet firms offering a wage given that a contact is made. If we call this element the matching probability,  $\check{z}(w)$ , that is, the probability of meeting a firm offering a wage  $w$ , we can rewrite [5] as:

$$\int_0^{\bar{w}} \lambda(w) \left[ \int_0^w \check{z}(w') x' \right] b z(w') x' \check{N} = \int_0^{\bar{w}} \check{z}(w) x' \check{N} \quad [6]$$

That is, inflows in firms offering a wage  $w$  must be equal to outflows from them in steady state. The inflows are given by the number of workers, unemployed or employed at a wage lower than  $w$ , that contact a given firm in moment  $x' \check{N}$  times the probability of meeting a firm offering exactly a wage  $w$ , given that the contact is made. The outflows from firms offering  $w$ ,  $b z(w)$ , in moment  $x' \check{N}$  is the proportion of their labor force,  $\lambda(w)$  which is either fired or contacts better paying firms. In equation [6] and hereafter we will assume, for the sake of brevity, that all offers are higher than the reservation wage, that is  $Z(w) = 0$  and we will refer to  $\bar{w}$  as the maximum offered wage.

We can make use here of the sampling theory to interpret the matching probability. In fact, the probability for a worker of matching with a firm offering a wage  $w$  is given by the following expression

$$\check{z}(w) = \frac{\lambda(w) b z(w)}{\int_0^{\bar{w}} \lambda(w') b z(w') x' \check{N}} \quad [7]$$

where  $z(w)$  is the density of firms offering  $w$  among all firms in the economy and  $\lambda(w)$  is the sampling probability, that is, the probability of contacting, in a given process of search, with a particular firm offering  $w$ . Again we define the support of  $Z(\cdot)$  as  $[\underline{w}, \bar{w}]$  being the minimum acceptable offered wage.

Each model assumes a particular way of sampling firms in the process of search, that is, a different sampling probability. This will lead not only to the corresponding matching probability,  $\check{z}(\cdot)$  but also to the equilibrium wage distribution function generated by the model.

In the case of the BM model, where it is assumed that matching is random, that is, the probability of sampling a firm offering  $\cdot$  is the same for all firms, we will have a sampling probability of  $\dagger(\cdot) = 1/\mathbb{Q}$ . Therefore, given (7), the matching probability is just the density of offered wages,  $\check{z}(\cdot) = z(\cdot)$ . Thus, the steady-state equality-of-flows condition in the BM model has the expression given by [5] in Section 2.

However, this specification is somewhat far from capturing the way in which one may think workers and firms match in the labor market. In fact, it seems natural to expect the probability of matching to depend on variables like the size of the firm (Burdett and Vishwanath, 1988), the type of contacts the worker has (Mortensen and Vishwanath, 1994), the firm's effort when searching for new workers (Robin and Roux, 1998), or the number of vacancies the firm is posting in the market. The first three ideas have been developed in different papers whose main results are going to be presented below. The last one is a new idea for trying to match the Wage Posting Approach with the Equilibrium Unemployment Approach. This is the same aim as in Mortensen (1998), but the goal here is to model more explicitly the matching probability.

All these ideas refer to a way of modeling the matching technology which is known as *balanced matching*. This alternative to random matching makes reference to the fact that the probability of sampling a given firm depends on its own characteristics. We will see how each model assumes a particular specification for the sampling probability, which results in a different matching probability.

### 3.1. *Balanced matching: the Burdett-Vishwanath model*

The model in Burdett and Vishwanath (1988) is not only different from BM in the sense that the matching technology changes, but also in other two main aspects. Firstly, workers choose their search intensity when they are looking for a job. This assumption makes the worker have a more precisely described behavior. However, as it does not play an essential role in the equilibrium solution, we will not focus on it. The second assumption refers to the firm's production technology,

which is not linear but increasing and concave in its workforce. Thus, there are decreasing returns to scale to labor. The following section of the paper deals with production technology assumptions; here I am going to highlight the results regarding the matching assumption. However, we also have to keep the former in mind because, as we will see afterwards, it is essential in order to obtain an equilibrium in this model. Moreover, both assumptions have to be modified at the same time if we want to obtain a new equilibrium.

With respect to the matching technology, it is assumed that a worker is more likely to contact larger, in terms of their workforce, than smaller firms. Specifically, it is assumed that the probability of sampling a firm offering a wage  $w$ ,  $\rho(w)$ , equals the number of workers employed by that firm divided by the total number of employed workers. That is:

$$\rho(w) = \frac{f(w)}{a \cdot i} N$$

Given the expression of the steady-state labor force of a firm offering  $w$ , equation [4], the matching probability,  $\tilde{z}(w)$  will be given by

$$\tilde{z}(w) = \frac{\frac{f(w)}{a \cdot i} b z(w)}{\int_{-\infty}^{\infty} \frac{f(y)}{a \cdot i} b z(y) x y} = \frac{f(w) b z(w)}{(a \cdot i) \int_{-\infty}^{\infty} \{y\} x y} = \frac{f(w) b z(w)}{a \cdot i} = \rho(w) \quad [8]$$

where, given that  $\{w\}$  is the density of earned wages, the integral in the denominator is equal to one.

Therefore, in this model, the probability of matching is not the probability of observing a firm offering  $w$  among the whole population but among employed workers. Hence, we can write the steady-state equality-of-flows condition for firms offering a wage  $w$ , equation (5), as follows:

$$\begin{aligned} & \rho(w) \cdot i \cdot (a \cdot i) \cdot \{w\} \cdot x \tilde{S} = \\ & \rho(w) \cdot i \cdot (a \cdot i) \cdot \{w\} \cdot x \tilde{S} \end{aligned} \quad [9]$$

where, making use of [4], we will have

$$\rho(w) \cdot i \cdot (a \cdot i) \cdot \{w\} = \rho(w) \cdot i \cdot (a \cdot i)$$

which implies that  $\lambda(w)$  must be constant and consequently, that the equilibrium distribution function of wages,  $Z(w)$ , must have a mass point. Hence, the result of a continuous distribution of wages obtained in the BM model is not obtained here any more.

The basic intuition behind this result can be captured by looking at the firm's decision problem. Now firms know that the probability of matching with workers is not constant but depends on the size of their workforce, which itself depends on the wage they post. Therefore, they will choose the wage which maximizes their steady-state profits subject to the equality-of-flows condition. Since all firms are equal, all of them will choose the same wage in equilibrium and therefore the only equilibrium in this game will be to post a wage which is equal to the value of the marginal productivity of the corresponding optimal steady-state labor force.

However, we should note that the assumption about the production technology is essential in this model in order to obtain the equilibrium result. We will see in the following section that without a decreasing returns to scale production function, the balanced matching assumption leads to the non-existence of an equilibrium in this model. Hence, both assumptions must be changed at the same time in order to obtain an equilibrium.

Furthermore, since in this model we have decreasing returns to scale to labor, for wage dispersion to be a possibility, it would be necessary to have a collection of optimal wages and workforce sizes yielding the same profit to the firm and, at the same time, verifying the steady-state equality-of-flows condition. This, as proved in Burdett and Vishwanath (1988), is not the case with balanced matching as modeled here.

However, one may think that this way of modeling matching is also *ad hoc*. Why should the matching probability depend on the size of the firm? Do workers really search more in larger than in smaller firms? We could think that, in fact, although the size of the firm is important, what really matters is how much effort a particular firm puts in recruiting through job offer advertising. The introduction of firms' effort in this literature has been carried out in Robin and Roux (1998), which will be presented later. But before doing so, let us first discuss an attempt to build a model which mixes the two usual ways of modeling matching, namely random and balanced matching.

3.2. *A mixture between random and balanced matching: The Mortensen-Vishwanath model*

Mortensen and Vishwanath (1994) highlights the fact that workers commonly use two different sources to get information about possible job offers: direct applications to employers and indirect contacts through friends and relatives. This allows them to obtain offers from a mixture of the distribution of wages offered by employers,  $Z(w)$ , and the distribution of wages earned by their personal contacts,  $Z^p(w)$ . In particular, workers will draw offers from  $\check{Z}[Z^p(w) + (1 - \check{Z})Z(w)]$ , with  $\check{Z}$  representing the fraction of offers received through personal contacts.

In fact, this model proposes a mixture between the cases of balanced and random matching. Here the matching probability is the weighted average of the probability for each case, that is:

$$\begin{aligned} \check{z}(w) &= \check{Z} \frac{\int_{\underline{w}}^{\infty} \frac{f(y)}{a} b z(y) dy}{\int_{\underline{w}}^{\infty} \frac{f(y)}{a} b z(y) dy} + (1 - \check{Z}) \frac{\int_{\underline{w}}^{\infty} \frac{1}{b} b z(y) dy}{\int_{\underline{w}}^{\infty} \frac{1}{b} b z(y) dy} \\ &= \check{Z}\{Z^p(w) + (1 - \check{Z})Z(w)\} \end{aligned} \quad [10]$$

Hence, in this model the steady-state equality-of-flows condition for firms offering a wage  $w$  is given by:

$$\begin{aligned} &[\theta_0 + \theta_1] [w^D(a - i)] (\check{Z}\{Z^p(w) + (1 - \check{Z})Z(w)\}) x \check{S} = \\ &[\theta_0 + \theta_1] (\check{Z}[Z^p(w) + (1 - \check{Z})Z(w)]) \cdot \frac{f(w)}{a} b z(w) x \check{S} \end{aligned} \quad [11]$$

where we can check that if  $\check{Z}$  is equal to 1 we return to the Burdett and Vishwanath (1988) model and if  $\check{Z}$  is equal to 0 we are again in the BM model.

The main result of the paper is that there exists a critical value of  $\check{Z}$  labeled  $\check{Z}^h$  such that for any value of  $\check{Z}$  below  $\check{Z}^h$  the steady-state equilibrium will be a dispersed distribution function of wage offers and for any value of  $\check{Z}$  above this threshold, there will exist a unique equilibrium where the offered wage is equal to the competitive one, that is, the value of marginal productivity. Therefore, depending on the fraction of offers a particular worker can obtain through personal contacts, she will be closer to the balanced or to the random matching case, with the corresponding results.

Although the basic idea in this paper helps to generalize the two basic approaches to matching, the way in which balanced matching is modeled suffers from the same criticisms as does Burdett and Vishwanath (1988). Therefore, we would like to have a more precise and accurate approximation to what really happens in the process of matching. A very recent and interesting idea is to model the hiring effort of firms looking for new employees. This idea is developed in Robin and Roux (1998) and it is presented in the following subsection.

3.3. *Balanced matching with firms' search effort: The Robin-Roux model*

Robin and Roux (1998) extends the BM model in different directions: firstly, it considers a balanced matching technology by introducing the firm's hiring effort in its decision problem. Secondly, the production function shows decreasing returns to scale in labor, and firms do not necessarily incorporate the same amount of capital. Therefore, they also model a firm's decision to enter the market, as involving a decision about capital. Although these two basic assumptions differ from the BM model, like Burdett and Vishwanath (1998), the way they are modeled is more precise and accurate.

In particular, the matching probability in this case takes into account not the firm's level of employment as a fraction of total employment, as in Burdett and Vishwanath (1988), but the level of hiring effort,  $y$ , devoted by the firm as a fraction of the total level of effort in the economy,  $Y$ . Using again our sampling theory terminology, the probability of sampling a given firm offering  $\theta$  and with a hiring effort  $y$  is given by this effort,  $y$  over the total level of effort,  $Y$ , and consequently, we will have that:

$$\tilde{z}(\theta, y) = \frac{y b(\theta, y) x^*(\theta, y)}{\int_{\underline{y}}^{\bar{y}} \int_{\underline{\theta}}^{\bar{\theta}} y b(\theta, y) x^*_1(\theta) x^*_2(y)} = \frac{y b(\theta, y) x^*(\theta, y)}{Y} \tag{12}$$

Here, since the strategy of the firm is twofold, we have to take into account the joint probability density function  $b(\theta, y)$  of  $\theta$  and  $y$  with respect to the product measure  $x^*(\theta, y) = x^*_1(\theta) \times x^*_2(y)$ .<sup>9</sup> Moreover, in

<sup>9</sup>In this model, the authors use general measures because they do not exclude the existence of mass points in the distribution of  $(\theta, y)$ . See Robin and Roux (1998) for a detailed exposition of these technical aspects.

this model the arrival rates of workers,  $\lambda_0$  and  $\lambda_1$ , are redefined to take into account the total hiring effort in the economy. They will be  $\lambda_0 Y$  and  $\lambda_1 Y$  instead of  $\lambda_0$  and  $\lambda_1 N$ . Therefore, we will have the following steady-state equality-of-flows condition

$$\begin{aligned} & [\lambda_0 Y i + \lambda_1 Y (1 - D)(a - i)] \frac{y b |(\cdot, \mathcal{Q}) x^*(\cdot, \mathcal{Q})}{Y} x \tilde{S} = \\ & \left[ (\lambda_0 + \lambda_1) Y \int_{\underline{y}}^{\bar{y}} \frac{y}{Y} b |(\cdot, \mathcal{Q}) x^*_1(\cdot) x^*_2(y) \right] \\ & \quad ,(\cdot, \mathcal{Q}) b |(\cdot, \mathcal{Q}) x^*(\cdot, \mathcal{Q}) x \tilde{S} \end{aligned} \tag{13}$$

where  $z(\cdot, \mathcal{Q})$  is the steady-state employment of a firm offering a wage  $\cdot$  and with a hiring effort  $y$ . Given that  $z(\cdot) = \int_{\underline{y}}^{\bar{y}} \frac{y}{Y} b |(\cdot, \mathcal{Q}) x^*_2(y) dy$  we will have that in steady-state  $z(\cdot, \mathcal{Q})$  verifies:

$$[\lambda_0 Y i + \lambda_1 Y (1 - D)(a - i)] \frac{Y}{Y} = [(\lambda_0 + \lambda_1) Y (1 - Z(\cdot))],(\cdot, \mathcal{Q}) N \tag{14}$$

Hence, with this way of modeling the matching technology, we are not able to conclude from the equality-of-flows condition itself whether the equilibrium distribution function of offered wages has a mass point or not. In fact, in this model any result can be obtained. In the article's Proposition 4, it is established that a different result can be obtained depending on the value of  $\lambda_1$ . The same crucial parameter as in the BM model, and of a new one: the cost of the hiring effort,  $w_0(y)$ . The possible equilibria are basically three: a distribution function with a mass point, a multiplicity of equilibrium distributions with or without mass points, and a unique continuous distribution function without mass points (see below).

The paper models the profit flow as follows. Given a level of capital,  $\epsilon$ , which is supposed to be chosen in a previous step, the steady-state profit flow will be

$$-\epsilon(\cdot, \mathcal{Q}) = \ddagger(\epsilon, (\cdot, \mathcal{Q})) \cdot z(\cdot, \mathcal{Q}) \cdot w_4(\cdot, \mathcal{Q}) \cdot w_0(y)$$

where  $\ddagger(\epsilon, (\cdot, \mathcal{Q})) = \ddagger(\epsilon, \mathcal{Q}(\cdot, \mathcal{Q}))$  is the production function, assumed to be Cobb-Douglas.  $w_4(\cdot, \mathcal{Q})$  is the cost of hiring the inflow  $z(\cdot, \mathcal{Q})$  of new workers,  $z(\cdot, \mathcal{Q}) = [\lambda_0 Y i + \lambda_1 Y (1 - D)(a - i)]$  and  $w_0(y)$  is

the cost of choosing a level  $y$  of hiring effort, both functions being increasing in their arguments.

The results of the paper are that if the offer arrival rate when employed,  $\lambda_1$  is equal to zero, all firms will choose the same wage, the minimum acceptable one, and that they will choose  $y$  in order to maximize profits given their previously chosen level of capital. However, if employees do receive alternative offers, it is not clear, as in the BM model, that a non-degenerate equilibrium distribution function exists. In fact, if posting  $y$  offers costs nothing, the argument of the BM model for avoiding mass points does not apply here: although  $\lambda_1(\cdot, \theta)$  increases at mass points, the hiring cost of these new workers,  $w(\theta)$  will increase as well. Thus, there is no possibility of deviating from a mass point equilibrium and, therefore, mass points can be found in equilibrium. Finally, if there exists a specific cost associated with the hiring effort, adjustment will not be free like before and the possibility of mass points in equilibrium disappears.

In this model, the role of the previously chosen level of capital is essential. As each firm has a different level of capital, the distribution function of wage offers,  $Z_\epsilon(\cdot)$  is dependent on capital and therefore the general distribution of wages in the market,  $Z(\cdot)$ , is the integrated value of this conditional distribution over the support,  $\underline{x}$ , of the capital distribution,  $\nu(\epsilon)$ , that is:

$$Z(\cdot) = \int_{\underline{x}} Z_\epsilon(\cdot) x \nu(\epsilon) N$$

To conclude, this model is a further step in formalizing the way in which firms and workers meet and match each other in the labor market. Furthermore it yields other richer results regarding, for example, the modeling of firm heterogeneity in terms of productivity differentials<sup>10</sup>.

However, in this paper the way in which balanced matching is modeled turns out to be a redefinition of the sampling probability assumed in Burdett and Vishwanath (1988),  $\lambda(\cdot) = \frac{\lambda_1(\cdot, \theta)}{a + \lambda_1(\cdot, \theta)}$  in the form  $\lambda = \frac{\lambda_1}{\lambda_1 + a}$  where now firms have twofold strategies  $(\cdot, \theta)$ .

<sup>10</sup> A new promising improvement of the BM model in the context of productivity dispersion *à la* Robin and Roux (1998) is considered in Postel-Vinay and Robin (1999). In this model firms counter the offers received by their employees from competing firms and try to yield no rent to their employees, in the sense that each worker is offered the minimum wage needed to attract her.

### 3.4. *Balanced matching in terms of the relative number of posted vacancies*

I end this section proposing another route closer to reality and with a clearer empirical counterpart, which has not been used yet in the context of the Wage Posting Approach<sup>11</sup>.

In terms of matching, what should really matter for firm to attract more workers is not its total size in terms of workers but the number of vacancies it is posting relative to the total number of vacancies in the economy. That is, if the matching probability is related to the relative number of vacancies the firm is posting, those firms with more vacancies should obtain more candidates for each of their vacant jobs.

If the firm wants to maintain its steady-state labor force, the number of vacancies it has to post in each period  $x\tilde{z}$  must be equal to the flow of workers who leave the firm in this period or, at least, a function of this outflow. Thus, taking the first strategy due to its simplicity, the number of vacancies  $\mathbb{C}^*$  for a firm offering  $w^*$  must be:

$$\mathbb{C}^*(w^*) = \left[ \frac{1}{\theta} + \theta \right] \int_{w^*}^{\infty} \tilde{z}(w) x^* dw, \quad (w^* | f(\mathbb{C}^*))N \quad [15]$$

The total number of vacancies in the economy will be called  $j$  and it is given by the integrated value of [15] over all possible values for wages. Therefore, if we assume that the sampling probability of a firm offering a wage  $w^*$  is the number of vacancies it posts over the total number of vacancies,  $\frac{\mathbb{C}^*(w^*)}{j}$  we will have that the matching probability in this case is:

$$\tilde{z}(w^*) = \frac{\frac{\mathbb{C}^*(w^*)}{j} b z(w^*)}{\int_{w^*}^{\infty} \frac{\mathbb{C}^*(w)}{j} b z(w) x^* dw} = \frac{\mathbb{C}^*(w^*) z(w^*)}{j} N \quad [16]$$

Hence, the steady-state equality-of-flows condition for this type of firms, given [15], will be:

<sup>11</sup>In a very recent paper entitled "Equilibrium unemployment with wage posting: Burdett-Mortensen meet Pissarides", Mortensen introduces the concept of vacancies in the Wage Posting Approach in order, as indicated by the title, to try to meet the Equilibrium Unemployment Approach. However, Mortensen assumes that random matching applies and the novelty with respect to the BM model is that now the arrival rates of job offers are functions of the aggregate level of vacancies in the economy.

$$[\theta_i + \lambda] \int_0^{\infty} (a - w) \frac{C(w)}{j} z(w) dw = C(w) b z(w) \tag{17}$$

Cancelling terms, we arrive to the same result as in Burdett and Vishwanath (1988): there must exist a mass point in  $\int_0^{\infty} (a - w) \frac{C(w)}{j} z(w) dw$  and therefore in the equilibrium wage distribution function.

Hence, although this way of modeling balanced matching is, from our point of view, more realistic, the basic result and its intuition are the same: if all firms and workers are respectively homogeneous and firms have influence on their matching probability, they are going to offer the same wage. Moreover, if there are decreasing returns to scale, that is, if each particular firm has an optimal level of employment, they will achieve it by posting the correct number of vacancies.

Finally, we can try to generalize this approach to balanced matching by using an even more general way of writing the matching probability, or, in the intermediate step, the sampling probability. We can think of these probabilities as given by some mixture between random and balanced matching. Specifically, we could assume that the sampling probability is something like  $\tilde{p}(w) = \tilde{\alpha} + \beta \frac{C(w)}{j}$  and, therefore, the matching probability will be given by  $\check{z}(w) = \left( \tilde{\alpha} + \beta \frac{C(w)}{j} \right) z(w)$  where  $\tilde{\alpha} = \frac{\tilde{\alpha}}{\tilde{\alpha} + \beta}$  and  $\beta = \frac{\beta}{\tilde{\alpha} + \beta}$  are positive parameters representing, respectively, some constant probability of sampling (random matching), and the efficiency with which balanced matching influences the matching probability of a given firm. Of course, one could use more general specifications but this simple linear way is capable of capturing the main features of the matching procedure.

Hence, we can again write the steady-state equality-of-flows equation for firms offering a wage  $w$ :

$$[\theta_i + \lambda] \int_0^{\infty} (a - w) \left( \tilde{\alpha} + \beta \frac{C(w)}{j} \right) z(w) dw = C(w) b z(w) \tag{18}$$

and using the definition of  $C(w)$ , equation [15], we could obtain the expression for the steady-state labor force,  $\int_0^{\infty} f(w) dw$  in this case.

With this general characterization of the matching probability, we do not arrive, in principle, to a situation which demands a mass point solution for the equilibrium wage distribution function. However, we

would have to solve the complete model in order to verify the specific characteristics of the solution. Nevertheless, this simple way of generalizing the matching probability leads to a better approximation to what the matching technology must be like in reality. Moreover, its advantage is that the concept of vacancies has a clear empirical counterpart, which will help the empirical implementation and estimation of the model.

#### 4. Decreasing returns to scale in the production function

As stated before, one assumption which seems quite unrealistic in the BM model is that the production function is linear in labor and therefore, there is no optimal level of labor in the firm.

Incorporating decreasing returns to scale was an early aim in the literature, for instance in Burdett and Vishwanath (1988). This paper was surveyed in the proceeding section because its main contribution concerns the matching technology. But it also uses a decreasing returns to scale production function and the result was already presented: no dispersed wage equilibrium exists. All firms offer the same wage, which is equal to the value of the marginal productivity of labor and, therefore, the equilibrium distribution function has a unique mass point at this wage. This result is mainly due to the assumption of balanced matching: as firms control the probability of matching, they choose the level of labor that allows them to pay the optimal wage.

As advanced in the previous section, the assumption of decreasing returns to scale is essential in order to obtain an equilibrium in this model. The assumed matching technology requires a nonlinear production function, because with a linear one no optimal level of employment different from zero would be found. Any candidate for equilibrium with a positive level of employment is not an equilibrium because firms can deviate from it and obtain more profits.

But we need to check what happens when matching between workers and firms is completely random. That is, maintaining one of the two key assumptions of the BM model and looking for the results changing the other one. This is done in Ridder and Van den Berg (1997). They assume random matching and that the production function is  $\lambda(n)$ , with  $n$  being the number of employees and  $\lambda(\cdot)$  being an increasing and concave function. Their main result is that there exists a wage  $w^h$  which is a mass point in the optimal distribution of wages above which

it is not optimal for firms to hire more workers. However, depending on the case, different results regarding the support of the optimal steady-state distribution function of offered wages can be obtained.

Ridder and Van den Berg (1997) takes the BM model as its baseline and simplifies it in the direction of the workers' search process. It assumes that the offer arrival rate,  $\lambda$ , is the same for unemployed and employed workers and, therefore, the reservation wage of unemployed workers is exactly their utility when unemployed,  $v$ . Moreover, the steady-state labor force of firms offering a wage  $w$  is given by

$$n(w) = \frac{a}{b} \frac{\lambda}{[1 + (\bar{Z}(w))] [1 + (\bar{Z}(w^D))]}$$

where  $\lambda = \lambda N$

As stated before, the main difference with the BM model is that the production technology,  $f(\cdot)$ , shows decreasing returns to scale to labor, and so the problem for the firm will be:

$$\max_N \{ a u^c - \lambda f(n) - v n \}$$

It can be shown that there exists a wage  $w^h = \lambda^{-1} f'(w^h)$  such that for firms paying  $w^h$  it is not optimal to increase their workforce. Moreover, we can obtain the following different equilibrium solutions depending on the value of the structural parameters  $\lambda, \alpha, \beta$  and the structural function  $f(\cdot)$ :

CASE I: *No production* when, at the minimum wage  $v$  profits are negative and firms prefer to have a smaller workforce than  $n(v)$ . This will happen because the firm's marginal productivity at this wage is lower than the cost per worker,  $v$ , and thus the firm would want to pay less than  $v$ . However, this is not possible given that  $v$  is the minimum acceptable wage. Moreover, at this level, profits are negative and therefore firms decide not to produce.

CASE II: *An unique mass point at  $b$*  when, at the minimum wage  $v$ , firms prefer to have a workforce smaller than  $n(v)$  although their profits are positive at that wage. That is, they will produce at the minimum wage but they will be restricted by this wage

in the sense that they would prefer a smaller workforce than  $\bar{v}$  because at this level the value of marginal productivity is lower than the marginal cost,  $v$ . However, they produce a positive amount in this case because profits are positive for wages equal to  $v$ .

CASE III: *An unique mass point at  $\bar{w} > v$*  In this case a wage higher than  $v$  is optimal and profits at wage  $\bar{w}$  are larger than those obtained with a dispersed equilibrium. Moreover, the value of the marginal productivity of  $\bar{v}$  is higher than the marginal cost,  $v$ . The firm wants to hire a larger workforce and thus it pays a higher wage. However, it is still more profitable for all firms to offer a single wage,  $\bar{w}$  and hence, we continue observing a mass point equilibrium. Note that here, as  $\bar{w} > v$  the workforce of each firm,  $\bar{n}(\bar{w})$  is bigger than in the previous cases.

CASE IV: *A positive density on  $[v, \bar{w}]$  and a mass point at  $\bar{w}$*  which will be obtained if it is optimal to offer wages equal to or higher than  $v$  and profits in the dispersed equilibrium case are larger than those obtained in a unique mass point equilibrium. Another condition to be fulfilled is that there must exist a solution inside the unit circle for the probability of having a mass point equilibrium,  $\bar{w}$ . On the equation which equalizes profits at the dispersed equilibrium and profits at the mass point  $\bar{w}$ .

CASE V: *A positive density on  $[v, \bar{w}]$  without mass points.* This case is obtained if the first two conditions in Case IV are fulfilled but there is no solution to the aforementioned equation inside the unit circle.

In these two final cases, the profits obtained by firms when they offer more than one wage have to be equal for each offered wage. That is, they must be equal to the profits obtained when the minimum wage,  $v$ , is offered. Therefore, in these two cases we are reproducing BM model's result, in the sense that firms are offering different wages and, as a consequence, they have different workforces. These are the five possible cases we can obtain with decreasing returns to scale.

There are other papers which also allow for decreasing returns to scale in the production function but which consider also some heterogeneity in firms' productivities. For example, Robin and Roux (1998) introduce this heterogeneity by assuming that firms first choose the level

of capital they will incorporate to the production process. They prove that, for each level of capital, the wage offered by each firm is unique. Moreover, there may exist a continuum of wages offered in the economy if the distribution of capital levels is continuous as well.

Lastly, we could introduce in a decreasing returns to scale environment not only firm’s heterogeneity in labor productivity, as in Robin and Roux (1998), but also some heterogeneity in the workforce. It would be very interesting to obtain results from distinguishing, for example, between skilled and unskilled workers in a non-segmented market and to find out what type of offers are made in equilibrium to each worker type.

In this and the preceding section we have studied the main consequences of relaxing the two basic assumptions of the BM model. Table 1 presents a summary of the main departures from this model, showing the main results obtained regarding the equilibrium wage distribution.

TABLE 1  
Main assumptions and results of various models

	Production Technology	Matching Technology	Worker Effort	Firm Effort.	Worker Heterog.	Firm Heterog.	Equilibrium Wage Distrib.
Burdett-Mortensen (1989, 1998)	linear	random	no	no*	no*	no	dispersed
Albercht-Axell (1984)	linear	random	no	no	yes	no	dispersed
Burdett-Vishwanath (1988)	DRS	balanced	yes	no	no	no	mas point
Mortensen-Vishwanath (1994)	DRS	balanced	no	no	no	no	both
Robin-Roux (1998)	DRS	balanced	no	yes	no	yes	both
Ridder-Van den Berg (1997)	DRS	random	no	no	no	no	both

DRS means decreasing returns to scale to labor.

\*The same results are obtained when both worker and firm heterogeneity are considered in this paper.

### 5. Structural estimation of wage posting models

The procedures for estimating partial equilibrium search models, where the distribution of wage offers is taken as given, are well known and developed (see Devine and Kiefer (1991) or Wolpin (1995) for extensive surveys). Here, we summarize a second-generation empirical literature. This literature estimates the determinants of reservation wages and the distribution of offered wages by using the results of Equilibrium Search Models. The objective here is not to present the specification of these structural estimations in detail, but only their main results<sup>12</sup>.

<sup>12</sup>For a broad exposition of the specification and the econometric techniques used in this literature, see Van den Berg (1999).

The empirical estimation strategy is often the same. By incorporating the structure of the theoretical model, the likelihood function for the observables of each worker in the sample is obtained and this is maximized with respect to the unknown parameters of the model, mainly job offer arrival rates, the separation rate, the value of unemployment time and the distribution of offered wages. The usual observable information used to write the likelihood function contains data on unemployment and employment spell durations and data on accepted and/or earned wages. Then, finding the theoretical relationships of these observables with the underlying parameters of the theoretical model, the latter can be estimated.

One of the first equilibrium search models appeared in Albrecht and Axell (1984). This model was estimated structurally in Eckstein and Wolpin (1990) using panel data on unemployment durations and reemployment wages for the US. The theoretical model deals with worker heterogeneity in their value of leisure and this feature is taken into account in the estimation by defining a finite number of worker types. However, since complexity in the computation of the equilibrium increases quickly with the number of worker types, only a small number of types can be considered. This strategy results in a poor estimate of the wage offer distribution function which, given the restriction imposed, is estimated to have a high percentage of measurement error. The reason is that each point in the support of the wage offer distribution necessarily equals the reservation wage of an unemployed worker type and it is imposed that these values are the only possible offered wages. One appealing result of these first models is that, due to the heterogeneity in the unemployed workers' value of leisure, they generate an unemployment duration distribution with negative duration dependence, which is in agreement with evidence from reduced-form studies. But their aforementioned results regarding the offered wage distribution function can only be considered as poor.

The ideas developed by Burdett and Mortensen and condensed in the BM model overcome certain problems regarding offered wage distribution functions. They are able to generate a continuous distribution where wages do not have to coincide with reservation wages. However, the main problem with the basic BM model with homogeneous workers and firms is that the equilibrium distribution of wage offers has an increasing density. This implication is at odds with observed wage distributions and, as discussed in the introduction, there is a need for

heterogeneity in order to match the model with the data. Thus, the estimation of this model has to deal with heterogeneity in firms and/or workers.

Kiefer and Neumann (1993), Koning, Ridder and Van den Berg (1995), and Ridder and Van den Berg (1998) estimate the basic BM model for the US, the first one, and for the Netherlands, the latter two, assuming that the labor market is segmented according to a set of observable or unobservable characteristics (education, industry, etc.). They estimate by maximum likelihood the structural parameters of the Equilibrium Search Model, by allowing them to vary across the submarket segments although all agents are respectively homogeneous within each submarket (this is called *between-market heterogeneity*). Most of the results in these papers are more in agreement with reality than those in Eckstein and Wolpin (1990) but there is one result which is not: the evolution of the wage earned by a given individual over her working life is quite limited, that is, the estimated return to experience is too small. One advantage of these between-market heterogeneity models is that they allow for the possibility of structural unemployment to occur in those segments where the minimum acceptable wage exceeds their productivity level. Thus, we can distinguish between *frictional* and *structural unemployment*. In Koning, Ridder and Van den Berg (1995) it is obtained that structural unemployment is particularly serious for teenagers. Moreover, they estimate that a increase in the minimum wage can increase the total structural unemployment rate by more or less one-for-one. Another interesting feature of these models is the possibility of decomposing wage variation into variation due to frictions, as in the BM model, and variation due to heterogeneity across segments. Typical results are that 50 % of wage variation is due to productivity dispersion and 25 % is due to search frictions. However, there is still more than 20 % of wage dispersion not explained by these two reasons. Bowlus, Kiefer and Neuman (1995, 1998) are the first to estimate Mortensen's (1990) model, a first version of the BM model with one labor market within which there exists firm heterogeneity in labor productivity (*within-market heterogeneity*) using US data. They assume a finite number of firm types and implement a likelihood procedure which involves the repeated computation of candidates for the equilibrium distribution function. As the computational complexity grows rapidly with the number of firm types, their results are only based on four or five points of support for the productivity distribution. However, the results are quite reasonable and interesting. In fact, their

estimated wage distribution fits pretty well the empirical one except for the lower tail, which is somewhat overestimated. Their estimates of the search friction parameters are about the same as those obtained with other procedures in other papers, which confirms the adequacy of their nonstandard estimation procedure.

The latest approach in the field of structural estimation of equilibrium search models is developed in Bontemps, Robin and Van den Berg (1999, 2000). These papers avoid the problem of computational complexity by assuming a continuous distribution of firm types. Their technique is based on using the first order conditions of the firm's problem and the one-to-one relationship between the wage offer and the productivity distribution functions. This technique results in joint estimates of the offer arrival rates, the separation rate, and the distribution of firms' productivities, the latter being estimated with a non-parametric technique. The empirical results of these papers, obtained with a sample of French data, are very satisfactory: they find that the estimated distribution function of offered wages, implied by the theoretical model, and the empirical wage distribution are quite consistent with one another. They also obtain that the most productive employers have significant monopsony power, which is used to pay wages much lower than the value of marginal product.

A further step is to incorporate within-market heterogeneity of individuals' value of leisure into these models. Burdett and Mortensen (1998) contains the theoretical analysis of this problem but it is in Bontemps, Robin and Van den Berg (1999) that this heterogeneity is firstly estimated. They use the same technique referred to above and allow for both firm and worker heterogeneity. They find that the majority of workers accept most job offers when unemployed and that the dispersion in the value of leisure for workers is not an important determinant of wage variations. However, these results are obtained under the maintained assumption of a unique offer arrival rate for both unemployed and employed workers. Unfortunately, the estimation without this assumption is extremely complicated and so they cannot carry it out, so as to confirm whether the previous results would also be obtained without it.

One of the latest structural estimations of wage posting models is carried out in Robin and Roux (1998). This paper allows for firm heterogeneity by introducing different levels of capital in the production process. Furthermore, they model the hiring process carried out by

firms with vacant jobs. They estimate the model using French firm data and obtain clear evidence of the importance of training and hiring costs. Their estimates for the elements of the search model are of the same magnitude as those found by Bontemps *et al.* (1999, 2000) using data for workers. This estimation, and the previous two are in the frontier of structural estimation of Equilibrium Search models with wage posting. They have succeeded in replicating the wage distribution observed in reality and in recovering the theoretical parameters of these models from data on unemployment durations and income.

Finally, one should note that none of these articles allows for heterogeneity in workers' productivities. Certainly, this is an extension which must be addressed in future because, as stated in Robin and Roux (1998), we need more than firm heterogeneity in order to fully match the wage heterogeneity observed in the data.

## 6. Concluding remarks

The present article surveys the recent literature on Equilibrium Search models with wage posting. The basic model is Burdett and Mortensen (1998). This is the most successful attempt to solve the so-called *Diamond Paradox*, that is, a degenerated equilibrium wage distribution function as a result of considering an equilibrium approach to job search. The BM model obtains wage dispersion in equilibrium by allowing workers to search on the job. However, this result is obtained under two maintained assumptions which can be questioned: firstly, the matching process is totally random, in the sense that each firm has an equal probability of matching with a given worker. Secondly, the production technology is linear in the workforce, that is, there exist constant returns to scale to labor.

The main conclusion of this study is that obtaining wage dispersion in equilibrium with homogeneous agents depends, basically, on these assumptions: if we substitute random matching by balanced matching, we obtain that the equilibrium is not dispersed but unique. It will be a mass point equilibrium at a wage equal to the value of the marginal productivity of the worker. However, we have also seen that this assumption cannot be removed without moving at the same time to a decreasing returns to scale production function. In fact, without this last assumption, there is no optimal level of employment when balanced matching is considered and therefore, the equilibrium does not exist.

Finally, we have pointed out a necessary further extension of these models in order to obtain a more complete description of wage variation. Namely, we have to deal with worker heterogeneity in terms of their productivity by, for example, introducing different worker types in the production function. The equilibrium search models that have been developed in the literature, almost always<sup>13</sup>, assume that there is no dispersion of worker-specific productivities. However, we know that there is also a worker-specific component in productivity which should be taken into account in Equilibrium Search models. This, as Van den Berg (1999) remarks, will be a promising “new avenue” for both theoretical and empirical Equilibrium Search models.

## Referencias

- Albrecht, J. W. and B. Axell (1994): “An equilibrium model of search unemployment”, *Journal of Political Economy* 92, pp. 824-840.
- Bontemps, C., J. M. Robin and G. J. Van den Berg (1999): “An empirical equilibrium job search model with search on the job and heterogeneous workers and firms”, *International Economic Review* 40, pp. 1039-1074.
- Bontemps, C., J. M. Robin and G. J. Van den Berg (2000): “Equilibrium search with continuous productivity dispersion: theory and non-parametric estimation”, *International Economic Review* 41, pp. 305-358.
- Bowlus, A. J., N. M. Kiefer and G. R. Neuman (1995): “Estimation of equilibrium wage distributions with heterogeneity”, *Journal of Applied Econometrics* 10, pp. S119-S131.
- Bowlus, A. J., N. M. Kiefer and G. R. Neuman (2001): “Equilibrium Search Models and the Transition from School to Work”, *International Economic Review* 42(2), pp. 317-343.
- Burda, M. and Ch. Wyplosz (1994): “Gross worker and job flows in europe”, *European Economic Review* 38, pp. 1287-1315.
- Burdett, K. and D. T. Mortensen (1989): “Equilibrium wage differentials and employer size”, Discussion Paper 860, Northwestern University.
- Burdett, K. and T. Vishwanath (1988): “Balanced matching and labor market equilibrium”, *Journal of Political Economy* 96, pp. 1048-1065.
- Burdett, K. (1990): “A new framework for labor market policy”, in J. Hartog, G. Ridder and J. Theeuwes (eds.), *Panel Data and Labor Market Studies*, pp. 297-312, North Holland, Amsterdam.
- Burdett, K. and D. T. Mortensen (1998): “Wage differentials, employer size and unemployment”, *International Economic Review* 39, pp. 257-273.
- Davis, S. J., and J. Haltiwanger (1990): “Gross job creation and destruction: microeconomic evidence and macroeconomic implications”, *NBER Macroeconomics Annual* 5, pp. 123-168.

<sup>13</sup>One exception is Manning (1993).

- Devine T. J. and N. M. Kiefer (1991), *Empirical Labor Economics: The Search Approach*, Oxford, University Press, New York.
- Diamond, P.A. (1971): "A model of price adjustment", *Journal of Economic Theory* 3, 156-168.
- Eckstein Z. and K. Wolpin (1990): "Estimating a market equilibrium search model from panel data on individuals", *Econometrica* 58, pp. 783-808.
- Gregg P. and A. Manning (1997): "Skill-biased change, unemployment and wage inequality", *European Economic Review* 41, pp. 1173-1200.
- Johnson G. and F. Sta/ord (1996): "Technology regimes and the distribution of real wages", Working Paper University of Michigan.
- Kiefer, N. M. and G. R. Neumann (1993): "Wage dispersion with homogeneity: the empirical equilibrium search model", in H. Bunzel et al.(eds.), *Panel Data and Labor Market Analysis, Panel Data and Labor Market Analysis*, pp. 57-74, North Holland, Amsterdam.
- Koning, P., G. Ridder and G. J. Van den Berg (1995): "Structural and frictional unemployment in an equilibrium search model with heterogeneous agents", *Journal of Applied Econometrics* 10, pp. S133-S151.
- Manning, A. (1993): "Endogenous labour market segmentation in a matching model", Discussion Paper 126, Centre for Economic Performance, London.
- Mortensen, D. T. (1986): "Job search and labor market analysis", in Ashenfelter, O. C. and Layard, R. (eds.), *Handbook of Labor Economics*, (Vol. II), pp. 849-919, North-Holland, Amsterdam.
- Mortensen, D. T. (1990): "Equilibrium wage distributions: a synthesis", in J. Hartog, G. Ridder and J. Theeuwes (eds.), *Panel Data and Labor Market Studies*, North Holland, Amsterdam.
- Mortensen D. T. (1998): "Equilibrium unemployment with wage posting: Burdett-Mortensen meet pissarides", CLS Working Paper 98-14.
- Mortensen, D. T. and G. R. Neuman (1988): "Estimating structural models of unemployment and job duration", in W.A. Barnett, E.R. Bernt, and H. White (eds.) *Dynamic Econometric Modelling, Proceedings of the Third International Symposium in Economic Theory and Econometrica*, Cambridge University Press, Cambridge.
- Mortensen D. T. and C. A. Pissarides (1994): "Job creation and job destruction in the theory of unemployment", *Review of Economic Studies* 61, pp. 397-415.
- Mortensen D. T. and C. A. Pissarides (1999): "New developments in models of search in the labor market", in O. Ashenfelter and D. Card (eds.), *Handbook of Labor Economics*, North-Holland, Amsterdam.
- Mortensen D. T. and T. Vishwanath (1988): "Information sources and equilibrium wage outcomes: it is who you know!", *Labour Economics* 1, pp. 187-201.
- Postel-Vinay F. and J. M. Robin (2002): "The distribution of earnings in an equilibrium search model with state-dependent o/ers and counter-o/ers", forthcoming in *International Economic Review*.

- Ridder, G. and G. J. Van den Berg (1997): "Empirical equilibrium search models", in D.M. Kreps and K.F. Wallis (eds.), *Advances in Economics and Econometrics: Theory and Applications*, Cambridge University Press, Cambridge.
- Ridder, G. and G.J. Van den Berg (1998): "An empirical equilibrium search model of the labor market", *Econometrica* 66, pp. 1183-1221.
- Robin, J. M. and S. Roux (1998): "Random or balanced matching: an equilibrium search model with endogenous capital and two-sided search", CREST Working Paper 9838.
- Sneessens H. and F. Shadman-Mehta (1995): "Skill demand and factor substitution", CEPR Discussion Paper 1279.
- Van den Berg, G. J. (1999): "Empirical inference with equilibrium search models of the labor market", *Economic Journal* 109, pp. F283-F306.
- Wolpin, K. I. (1995), *Empirical Methods for the Study of Labor Force Dynamics*, Harwood Academic Publishers, Luxembourg.

## Resumen

*Este artículo presenta un resumen de la literatura más reciente sobre el enfoque de Anuncio Salarial en los Modelos de Búsqueda de empleo en Equilibrio. Partiendo del modelo básico de Burdett y Mortensen (1998), se describen las principales consecuencias de la no consideración de sus dos principales supuestos: emparejamiento aleatorio y función de producción lineal. Se muestran como la modelización concreta de tanto la función de producción como la de emparejamientos puede afectar a los resultados que se obtienen para la función de distribución de los salarios ofrecidos. En una segunda parte, se introducen y discuten los principales resultados empíricos de la estimación estructural de estos modelos.*

*Palabras clave: Modelos de búsqueda de empleo en equilibrio, emparejamiento aleatorio o proporcional, rendimientos de escala.*

*Recepción del original, junio de 2000  
Versión final, julio de 2001*