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RISK AND RETURN IN THE SPANISH STOCK MARKET: SOME EVIDENCE FROM INDIVIDUAL ASSETS

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We use monthly returns on individual shares in the Spanish stock market between 1963 and 1992 to test the restrictions that dynamic APT models impose on the risk-return relationship.

We find systematic biases in average risk premia when a value weighted index is used as observable factor, but not when the factor is left unspecified. When we model the time-variation in the covariance matrix of returns, we find that the "price of risk" is not common across assets whether or not the factor is prespecified. The results are robust to the presence or absence of a riskless asset. (*JEL* G12)

1. Introduction

Empirical tests of asset pricing theories have traditionally focused on their cross-sectional implications by checking if the models could explain differences in asset returns, at least in terms of temporal averages. For the Spanish case, for instance, Palacios (1973), Berges (1984), Rubio (1988) and Gallego, Gómez and Marhuenda (1992) test, without much success, whether there is a positive linear relationship between the average return on an asset over time and the unconditional covariance of that asset with the market portfolio, as

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postulated by the static version of the Capital Asset Pricing Model (CAPM)

Recently, the emphasis has shifted towards intertemporal asset pricing models in which agents actions are based on the distribution of returns conditional on the available information, which is obviously changing. This is partly motivated by the fact that, nowadays, it is well documented and widely recognized that the volatility of financial markets changes over time. Besides, the new approach has had some empirical success. For instance, Ng, Engle and Rothschild (1992) found that, unlike in a static setting, the basic restrictions of a CAPM-type model were not rejected with US data when they allowed the variances and covariances of the assets to vary over time.

At the same time, there has been a renewed interest in studying the temporal variation in the volatility of the Spanish stock market (e.g. Alonso, 1995, Peiró, 1992, or Peña and Ruiz, 1993). Nevertheless, these studies are mainly descriptive and have only considered the market index, with some exceptions, like Alonso and Restoy (1995), who study the risk-return relationship for the Spanish portfolio of the Morgan-Stanley database.

Therefore, it seems appropriate to combine both strands of the literature, and study the valuation of risk in the Spanish stock market at a disaggregate level by testing dynamic asset pricing models which explicitly allow for time-variation in the variances and covariances of the assets. But this requires a multivariate model of the time-varying covariance matrix of returns. In the absence of any structure, the estimation of such a model would entail the parametrisation of time series processes for each of the distinct elements of the covariance matrix, which is clearly intractable for even a moderately large number of assets. It is perhaps not surprising that one of the most popular approaches to multivariate dynamic heteroskedasticity employs the same idea as traditional factor analysis to obtain a parsimonious representation of conditional second moments. That is, it is assumed that the innovations in each of the assets are a linear combination of a small number of common factors plus an idiosyncratic noise term, but allowing for conditional heteroskedasticity-type effects in the underlying factors. Such models are particularly appealing in our context, since the concept of factors plays a fundamental role is major asset pricing models, such as the Arbitrage Pricing Theory (APT) of Ross' (1976).

In Sentana (1995a), we employed such an approach for monthly return data on ten size-ranked portfolios, and found that the asset pricing restrictions did not appear to be sustained. As a possible explanation, we conjectured that the behaviour of stock returns on small firms, which tend to have low volume and frequency of trading, could account for the rejection of the model. Here, we shall repeat the exercise with data on individual stocks listed in the Spanish stock market between January 1963 and December 1992. As theoretical model, we shall use a generalization of the dynamic version of the APT developed in King, Sentana and Wadhwani (1994), which does not require the existence of a safe asset. We use modern intertemporal asset pricing theory to rigorously derive a general version of this model in Section 2. We also discuss the implications of simplifying assumptions, such as constant factor loadings and risk prices, or univariate GARCH parametrisations, made for empirical tractability. The econometric methodology is explained in detail in Section 3, while a description of the data can be found in Section 4. The main empirical results are discussed in Section 5. Finally, Section 6 contains the conclusions.

2. Theoretical Model

2.1. Intertemporal asset pricing

Our model is formally based in an economy with a countably infinite collection of primitive assets, whose payoffs are defined on an underlying probability space. Let R_{it} $(i = 1, 2, \ldots, N, \ldots)$ be the random (gross) holding return from a unit investment in asset i during period t, which is uncertain from the agents' point of view at time $t-1$ since the asset is risky. Importantly, the analysis is carried out in terms of the conditional distribution of period t returns, where the relevant information set, I_{t-1}, contains at least the past values of asset returns.

Let L_t^2 denote the collection of all random variables defined on the underlying probability space which are measurable with respect to I_t and have finite conditional second moments. Hansen and Richard (1987) show that L_t^2 is the conditional analogue of a Hilbert space under the conditional mean square inner product, $E(p_t q_t \mid I_{t-1})$, and the associated norm, $\|p_t\|_{t-1} = E^{1/2}(p_t^2 \mid I_{t-1})$, where $p_t, q_t \in L_t^2$. We assume that, conditional on I_{t-1}, the second moments of the primitive assets are bounded (so that $R_{it} \in L_t^2$), although their unconditional second moments may be unbounded (cf. Engle and Bollerslev, 1986, IGARCH processes). In addition, we also assume that the conditional covariance matrix of any sequence of asset returns is positive definite, so that no primitive asset is redundant.

Let u_{it} be the unanticipated (as of time $t-1$) component of asset i returns, i.e. $u_{it} = R_{it} - \nu_{it}$, where $\nu_{it} = E(R_{it} \mid I_{t-1})$ and let $\sigma_{ijt} = E(u_{it}u_{jt} \mid I_{t-1}) = cov(R_{it}, R_{jt} \mid I_{t-1})$. The basic assumption made on the stochastic structure of returns for the primitive assets is that u_{it} has a conditional factor representation, so that we can write:

$$u_{it} = \beta_{i1t} f_{1t} + \beta_{i2t} f_{2t} + \ldots + \beta_{ikt} f_{kt} + v_{it} \quad (i = 1, 2, \ldots, N, \ldots) \quad [1]$$

where f_{jt} ($j = 1, 2, \ldots, k$ finite) are common factors which capture systematic risk affecting all assets, $\beta_{ijt} \in I_{t-1}$ ($i = 1, 2, \ldots, N, \ldots; j = 1, 2, \ldots, k$) are the associated factor loadings known in $t-1$ which measure the sensitivity of the asset to the common factors, while v_{it} are idiosyncratic terms reflecting risks specific to asset i, which by definition are (conditionally) orthogonal to the common factors. It is important to stress that, in general, the factor loadings change from asset to asset.

We assume (without loss of generality at the theoretical level) that the common factors represent conditionally orthogonal influences. To guarantee that common and specific factors are innovations, we also assume that they are unpredictable on the basis of I_{t-1}. Nevertheless, their conditional variances, λ_{jt} ($j = 1, \ldots, k$) and ω_{iit} ($i = 1, \ldots$), which reflect their volatility, may change over time in a predictable manner. In this way, it is possible to allow for short and long periods of low and high volatility, both for all assets simultaneously, and for each one separately. Note also that the conditional correlation coefficients will generally change over time. For instance, increases in the volatility of a factor that affects two assets with the same sign will be associated with an increase in their correlation, while increases in idiosyncratic volatility will produce the opposite effect (see King, Sentana and Wadhwani, 1994).

Finally, we allow idiosyncratic terms to be conditionally correlated, but only mildly so in order to guarantee that full diversification applies (see Chamberlain and Rothschild, 1983, for a precise characterization of the required condition). Notice that [1] is then a statement about the cross-sectional dependence of asset returns.

In this framework, any collection of N asset returns can be represented as:

$$\mathbf{R}_{Nt} = \boldsymbol{\nu}_{Nt} + \mathbf{u}_{Nt} = \boldsymbol{\nu}_{Nt} + \mathbf{B}_{Nt}\mathbf{f}_t + \mathbf{v}_{Nt} \qquad [2]$$

where $E(\mathbf{u}_{Nt} \mid I_{t-1}) = \mathbf{0}$ and $V(\mathbf{u}_{Nt} \mid I_{t-1}) = \mathbf{\Sigma}_{Nt} = \mathbf{B}_{Nt}\mathbf{\Lambda}_t\mathbf{B}'_{Nt} +$ $_{Nt}$, with $[\mathbf{R}_{Nt}]_i = R_{it}, [\boldsymbol{\nu}_{Nt}]_i = \nu_{it}, [\mathbf{u}_{Nt}]_i = u_{it}, [\mathbf{v}_{Nt}]_i = v_{it}, [\mathbf{B}_{Nt}]_{ij} = \beta_{ijt}, [\mathbf{\Lambda}_t]_{jj} = \lambda_{jt}$ and $[\quad_{Nt}]_{ij} = cov(v_{it}, v_{jt} \mid I_{t-1}) = \omega_{ijt}$.

Let $p_{Nt} = \sum_{i=1}^{N} w_{it}R_{it} = \mathbf{w}'_{Nt}\mathbf{R}_{Nt}$ be the payoff of a portfolio of N primitive assets with weights $\mathbf{w}_{Nt} \in I_{t-1}$, and define its cost, $C(p_{Nt})$, as the sum of its weights, $\sum_{i=1}^{N} w_{it} = \mathbf{w}'_{Nt}\ell_N$. If we call P_{Nt} the set of payoffs from all possible portfolios of the N primitive assets with weights determined in $t-1$, the conditional mean-variance (MV) frontier generated from $\mathbf{R}_{Nt}$ is defined as the set of portfolios in P_{Nt} which solve the program $\min V(p_{Nt} \mid I_{t-1})$ subject to $C(p_{Nt}) = 1$ and $E(p_{Nt} \mid I_{t-1}) = \bar{\nu}_t$, for $\bar{\nu}_t \in I_{t-1}$.[1] If not all ν_{it} are equal,[2] it is possible to prove that such a conditional MV frontier can be spanned by two (conditionally) orthogonal portfolios: the minimum conditional second moment unit-cost portfolio, with return $R^*_{Nt} = \mathbf{w}^{*\prime}_N\mathbf{R}_{Nt}$, and an arbitrage (i.e. zero-cost) portfolio, with payoff $p^{\#}_{Nt} = \mathbf{w}^{\#\prime}_N\mathbf{R}_{Nt}$,

[1] The usual dual interpretation of conditional moments as real numbers or random variables also applies to conditional MV frontiers. In the former case, we have one frontier for each possible value of the conditioning variables, whereas in the latter, the frontier itself is a random function measurable with respect to I_{t-1}.

[2] Otherwise, the frontier collapses to a single point: the minimum (conditional) variance portfolio $\ell'_N\mathbf{\Sigma}^{-1}_{Nt}\mathbf{R}_{Nt}/(\ell'_N\mathbf{\Sigma}^{-1}_{Nt}\ell_N)$.

where:

$$\begin{aligned}
\mathbf{w}^*_{Nt} &= \frac{(\boldsymbol{\Sigma}_{Nt}+\boldsymbol{\nu}_{Nt}\boldsymbol{\nu}'_{Nt})^{-1}\ell_N}{\ell'_N(\boldsymbol{\Sigma}_{Nt}+\boldsymbol{\nu}_{Nt}\boldsymbol{\nu}'_{Nt})^{-1}\ell_N} \\
&= \frac{1+\text{B}_{Nt}}{\text{C}_{Nt}+\text{D}_{Nt}}(\boldsymbol{\Sigma}^{-1}_{Nt}\ell_N) - \frac{\text{A}_{Nt}}{\text{C}_{Nt}+\text{D}_{Nt}}(\boldsymbol{\Sigma}^{-1}_{Nt}\boldsymbol{\nu}_{Nt}) \\
\mathbf{w}^{\#}_{Nt} &= (\boldsymbol{\Sigma}_{Nt}+\boldsymbol{\nu}_{Nt}\boldsymbol{\nu}'_{Nt})^{-1}\boldsymbol{\nu}_{Nt} \\
&\quad -\frac{\ell'_N(\boldsymbol{\Sigma}_{Nt}+\boldsymbol{\nu}_{Nt}\boldsymbol{\nu}'_{Nt})^{-1}\boldsymbol{\nu}_{Nt}}{\ell'_N(\boldsymbol{\Sigma}_{Nt}+\boldsymbol{\nu}_{Nt}\boldsymbol{\nu}'_{Nt})^{-1}\ell_N}(\boldsymbol{\Sigma}_{Nt}+\boldsymbol{\nu}_{Nt}\boldsymbol{\nu}'_{Nt})^{-1}\ell_N \\
&= \frac{\text{C}_{Nt}}{\text{C}_{Nt}+\text{D}_{Nt}}(\boldsymbol{\Sigma}^{-1}_{Nt}\boldsymbol{\nu}_{Nt}) - \frac{\text{A}_{Nt}}{\text{C}_{Nt}+\text{D}_{Nt}}(\boldsymbol{\Sigma}^{-1}_{Nt}\ell_N)
\end{aligned}$$

with $\text{A}_{Nt}=\ell'_N\boldsymbol{\Sigma}^{-1}_{Nt}\boldsymbol{\nu}_{Nt}$, $\text{B}_{Nt}=\boldsymbol{\nu}'_{Nt}\boldsymbol{\Sigma}^{-1}_{Nt}\boldsymbol{\nu}_{Nt}$, $\text{C}_{Nt}=\ell'_N\boldsymbol{\Sigma}^{-1}_{Nt}\ell_N$ and $\text{D}_{Nt}=\text{B}_{Nt}\text{C}_{Nt}-\text{A}^2_{Nt}$.[3] Therefore, in (conditional) MV space (conditional) frontier portfolios lie on the parabola:

$$\{\bar{\nu}_t; \left[E^{-1}(p^{\#}_{Nt}\mid I_{t-1})-1\right]\bar{\nu}^2_t - 2\left[E(R^*_{Nt}\mid I_{t-1})E^{-1}(p^{\#}_{Nt}\mid I_{t-1})\right]\bar{\nu}_t$$

$$+\left[E(R^{*2}_{Nt}\mid I_{t-1})+E^2(R^*_{Nt}\mid I_{t-1})E^{-1}(p^{\#}_{Nt}\mid I_{t-1})\right]\} =$$

$$[\bar{\nu}_t; (\text{C}_{Nt}/\text{D}_{Nt})\bar{\nu}^2_t - 2(\text{A}_{Nt}/\text{D}_{Nt})\bar{\nu}_t + (\text{B}_{Nt}/\text{D}_{Nt})]$$

(cf. Huang and Litzenberger, 1988).

If we expand P_{Nt} into P_{N+1t} by adding another primitive asset to the N assets under consideration, for each value of $\bar{\nu}_t$ we obtain a new frontier portfolio whose conditional variance is at least as small as before. In principle, though, we do not know what the limit to this process is. Therefore, it is important to extend mean-variance analysis to *all* primitive assets. As we shall see, such an extension, leaves the results largely unaffected but requires some extra notation.

Let $\bar{P}_t$ be the conditional closure of the set of payoffs from all possible portfolios of the primitive assets. It is clear that $\bar{P}_t$ is a conditionally

[3] The (conditional) MV frontier can be spanned in infinitely many other ways. For instance, Huang and Litzenberger (1988) use as spanning portfolios $\mathbf{w}^{g\prime}_N\mathbf{R}_{Nt}$ and $\mathbf{w}^{h\prime}_N\mathbf{R}_{Nt}$, where $\mathbf{w}^g_{Nt}=\mathbf{w}^*_{Nt}-(\text{A}_{Nt}/\text{D}_{Nt})\mathbf{w}^{\#}_{Nt}$ and $\mathbf{w}^h_{Nt}=[(\text{C}_{Nt}+\text{D}_{Nt})/\text{D}_{Nt}]\mathbf{w}^{\#}_{Nt}$. Alternatively, Chamberlain and Rothschild (1983) use $\mathbf{w}^*_{Nt}$ and $\mathbf{w}^+_{Nt}=\mathbf{w}^*_{Nt}+[(1+\text{B}_{Nt})/\text{A}_{Nt}]\mathbf{w}^{\#}_{Nt}=(1/\text{A}_{Nt})(\boldsymbol{\Sigma}^{-1}_{Nt}\boldsymbol{\nu}_{Nt})$.

closed linear subspace of L_t^2, and hence also a conditional analogue to a Hilbert space under the (conditional) mean square inner product. We assume enough restrictions on (β_{ji}, ν_{it}) to guarantee that it is possible to mimic (in the conditional mean square norm) the behaviour of the common factors by means of elements of $\bar{P}_t$ (see Chamberlain and Rothschild, 1983, and Sentana, 1994a).

In order to model conditional mean returns, we shall use the cost functional $C(.)$, which can be regarded as a linear pricing functional defined on $\bar{P}_t$ that maps elements of this space onto the information set, I_{t-1}. Under a mild no arbitrage condition (see Hansen and Richard, 1987), this functional is conditionally continuous on $\bar{P}_t$. Then, a conditional version of the Riesz representation theorem implies that there is a portfolio with payoff $p_t^* \in \bar{P}_t$, such that $C(p_t) = E(p_t^* p_t \mid I_{t-1})$ for all $p_t \in \bar{P}_t$, i.e. p_t^* represents $C(.)$ in $\bar{P}_t$ (see Hansen and Richard, 1987). We can interpret p_t^* as a stochastic discount factor which prices the available assets by discounting their uncertain payoffs across different states of the world. We shall also use the conditional mean functional, $E(. \mid I_{t-1})$, which is always conditionally continuous on $\bar{P}_t$, and therefore can also be represented by a portfolio with payoff $p_t^+ \in \bar{P}_t$, so that $E(p_t \mid I_{t-1}) = E(p_t^+ p_t \mid I_{t-1})$ for all $p_t \in \bar{P}_t$. Let $R_t^* = p_t^*/C(p_t^*)$ be the return on a unit investment on the pricing representing portfolio, $R_t^+ = p_t^+/C(p_t^+)$ the corresponding return for the conditional mean representing portfolio, and define $p_t^\# = C(p_t^+)[R_t^+ - R_t^*]$. Then, if we rule out risk neutral pricing, so that not all conditional expected returns are equal, it is possible to prove that the returns on portfolios on the conditional MV frontier generated from all primitive assets satisfy the relationship $R_t = R_t^* + \gamma_t p_t^\#$, where $\gamma_t = [\bar{\nu}_t - E(R_t^* \mid I_{t-1})]/E(p_t^\# \mid I_{t-1})$ (see Hansen and Richard, 1987). Hence, in (conditional) MV space, (conditional) frontier portfolios for $\bar{P}_t$ lie on the parabola:

$$\{\bar{\nu}_t; \left[E^{-1}(p_t^\# \mid I_{t-1}) - 1\right] \bar{\nu}_t^2 - 2\left[E(R_t^* \mid I_{t-1})E^{-1}(p_t^\# \mid I_{t-1})\right] \bar{\nu}_t$$
$$+ \left[E(R_t^{*2} \mid I_{t-1}) + E^2(R_t^* \mid I_{t-1})E^{-1}(p_t^\# \mid I_{t-1})\right]\}$$

which is a hyperbola in (conditional) mean-standard deviation space. If there is a limit portfolio which is conditionally riskless (see Chamberlain and Rothschild, 1983, for a necessary and sufficient condition), the hyperbola coincides with its asymptotes, and the portfolio frontier is a reflected straight line which passes through the riskless portfolio and R_t^*.

In this framework, it is possible to prove that there is a conditionally affine relationship between expected returns and betas with respect to any portfolio on the conditional MV frontier other than the minimum conditional variance portfolio (see Hansen and Richard, 1987). In particular, if R_{et} denotes the return on a conditional frontier portfolio, we have

$$\nu_{it} - \nu_{0t} = \beta_{iet}(\nu_{et} - \nu_{0t}) \quad [3]$$

where $\nu_{et} = E(R_{et} \mid I_{t-1}), \beta_{iet} = cov(R_{it}, R_{et} \mid I_{t-1})/V(R_{et} \mid I_{t-1})$, and ν_{0t} is the expected return on the corresponding zero-beta portfolio, i.e. a frontier portfolio whose conditional covariance with R_{et} is zero. When there is a conditionally riskless portfolio, ν_{0t} must be equal (in conditional mean square) to its (gross) return, and $\nu_{it} - \nu_{0t}$ becomes the risk premium on asset i, μ_{it}, which is also the conditional expected value for excess returns, $r_{it} = R_{it} - R_{0t}$. In what follows, we shall refer to $\mu_{it} = \nu_{it} - \nu_{0t}$ and $R_{it} - \nu_{0t}$ as "risk premium" and "excess returns" respectively, even when no riskless asset exists.

Our crucial asset pricing assumption is that there is a portfolio on the conditional MV frontier generated from all primitive assets which contains only factor risk.[4] More formally, if R_{et} denotes its return, we assume that:

$$R_{et} = \nu_{et} + \beta_{e1t}f_{1t} + \beta_{e2t}f_{2t} + \ldots + \beta_{ekt}f_{kt} \quad [4]$$

If R_{et} is the return on the market portfolio, equation [3] above coincides with a conditional version of the CAPM in which, irrespectively of the number of factors, risk premia are proportional to the conditional covariance of each asset with the market.

[4]Otherwise, it is only possible to prove that the pricing error of expression [6] below would be negligible on average cross-sectionally, but not necessarily so for each asset (see Chamberlain, 1983). Note, however, that we are not assuming that the whole conditional MV frontier is well-diversified, unless there is a riskless asset (cf. Huberman and Kandel, 1987).

When R_{et} is not observable, though, expression [3] has to be expanded further. We know that:

$$\beta_{iet} = \frac{\beta_{e1t}\lambda_{1t}\beta_{i1t} + \ldots + \beta_{ekt}\lambda_{kt}\beta_{ikt}}{\beta_{e1t}^2\lambda_{1t} + \ldots + \beta_{ekt}^2\lambda_{kt}}$$

Let $R_{f_1t}, \ldots, R_{f_kt}$ be the returns on k limiting factor representing portfolios, i.e. unit cost well-diversified portfolios of risky assets which have unit loading on only one factor and zero loadings on the other factors. Then

$$E(R_{f_jt} \mid I_{t-1}) - \nu_{0t} = \frac{\beta_{ejt}(\nu_{et} - \nu_{0t})}{\beta_{e1t}^2\lambda_{1t} + \ldots + \beta_{ekt}^2\lambda_{kt}}\lambda_{jt} = \tau_{jt}\lambda_{jt} = \pi_{jt} \quad [5]$$

If combine both expressions we get:

$$\begin{aligned} \mu_{it} &= \nu_{it} - \nu_{0t} = \beta_{i1t}\tau_{1t}\lambda_{1t} + \ldots + \beta_{ikt}\tau_{kt}\lambda_{kt} \\ &= \beta_{i1t}\pi_{1t} + \ldots + \beta_{ikt}\pi_{kt} \end{aligned} \quad [6]$$

Therefore, "risk premia" can be written as an exact linear combination of the volatility of the common factors, with weights proportional to the corresponding factor loadings. This expression reduces to expression [7] in King, Sentana and Wadhwani (1994) if there is a riskless asset. The reason is that in that case, our assumption that there is a portfolio on the conditional MV frontier generated from all primitive assets which only contains factor risk is equivalent to their assumption that the pricing representing functional, p_t^*, is well diversified.

An important feature of [6] is that it provides a connection between conditional means of "excess returns", or "risk premia", and their conditional variances and covariances, which measure their volatility and covariation. As we shall see below, such a relationship is very convenient for estimation purposes.

Equation [6] above can also be interpreted as saying that the "risk premium" on an asset is a linear combination of its factor loadings or betas, with weights common to all assets. As usual, the common weights, $\pi_{jt} = \lambda_{jt}\tau_{jt}$, can be understood as the "risk premium" on the j^{th} (limiting) factor mimicking portfolio. Therefore, we can also say that asset risk premia are linear combinations of k risk premia associated with the common factors. In that sense, expression [6] above could also be obtained with a conditional version of Ross (1976) exact APT. Given that λ_{jt} is the volatility of both factor j and its representing portfolio, and π_{jt} the "risk premium" on that portfolio, we can interpret $\tau_{jt} = \pi_{jt}/\lambda_{jt}$ as the "price of risk" for that factor. That is, the amount of expected return that agents would be willing to give away to reduce its variability by one unit.

It is important to emphasize that, according to the model, risk prices depend on the factors, not on the assets, since otherwise there would be arbitrage opportunities. Furthermore, the model also implies that specific risk, as measured by the volatility of the idiosyncratic terms, should not be priced because it can be diversified away. Its price, thus, should be zero. These fundamental restrictions shall be tested.

The model to be estimated is then

$$\begin{aligned} R_{it} &= \nu_{0t} + \beta_{i1t}\lambda_{1t}\tau_{1t} + \ldots + \beta_{ikt}\lambda_{kt}\tau_{kt} + \\ &\quad + \beta_{i1t}f_{1t} + \ldots + \beta_{ikt}f_{kt} + v_{it} \\ &= \nu_{0t} + \beta_{i1t}(\lambda_{1t}\tau_{1t} + f_{1t}) + \ldots + \beta_{ikt}(\lambda_{kt}\tau_{kt} + f_{kt}) + v_{it} \\ &= \nu_{0t} + \beta_{i1t}(R_{f_1 t} - \nu_{0t}) + \ldots + \beta_{ikt}(R_{f_k t} - \nu_{0t}) + v_{it} \qquad [7] \end{aligned}$$

One problem with the above expression is that it does not necessarily price derivative assets correctly. Nevertheless, it can be safely applied to portfolios of the primitive assets. Consider for simplicity the case of a single common factor. Let R_{pt} be the excess return on a portfolio with weights $\mathbf{w}_{pt}$ (known in period $t-1$). Our assumptions imply that we can write R_{pt} as:

$$R_{pt} - \nu_{0t} = \beta_{p1t}(\lambda_{1t}\tau_{1t} + f_{1t}) + v_{pt} \qquad [8]$$

where the factor loading coefficient, β_{p1t}, and the specific risk component, v_{pt}, are a linear combination of the individual $\beta'_{i1t}s$ and $v'_{it}s$, but the common factor, its variance and price of risk are the same as in equation [6].

From an economic point of view, the hypothesis of interest are the following:

Are the risk prices different from zero?

Are the same risks valued in the same way across assets?

Is idiosyncratic risk priced?

Are there systematic biases in risk premia which cannot be explained by the model, such as "size effects"?

But before, we have to transform equation [6] above, which is a period by period cross-sectional restriction on the relative pricing of any subset of assets, into an estimable model of the time-variation in risk premia. For this purpose, assumptions about the evolution of $\beta_{ijt}, \tau_{jt}, \lambda_{jt}$, and ω_{ijt} are necessary.

2.2. Conditional moments specification

Our first simplifying assumption is that the idiosyncratic terms of the assets at hand are conditionally orthogonal to each other, so that $_{Nt}$ is actually diagonal. Therefore, we are assuming a conditionally orthogonal exact k factor structure (cf. Chamberlain and Rothschild, 1983). Second, we assume that, for any given unconditional normalization of the factors, the factor loadings and the prices of risk are time-invariant. Such an assumption is observationally equivalent to a model in which the conditional variance of the factors is constant, but the betas of different assets on a factor change proportionately over time.

Finally, we need to specify the temporal variation in the volatility of common and idiosyncratic factors to complete the model. In practice, we shall assume that such variances can be modelled as *univariate* GARCH-type processes. In particular, we assume that they follow the GQARCH(1,1) (quadratic GARCH) model in Sentana (1995b). This model not only captures the autocorrelation in stock market volatility, but also allows for asymmetric effects in the response of volatility to positive and negative shocks of the same size, and has been successfully applied to US data (see Campbell and Hentschel, 1992) and UK data (see e.g. Demos, Sentana and Shah, 1993). Given [5], and the constancy of τ, this assumption implies that the factor representing portfolios $R_{f_1 t}, \ldots, R_{f_k t}$ will follow univariate GQARCH(1,1)-M models.

Importantly, if the unconditional variances of the factors and idiosyncratic disturbances are bounded, our assumptions also have implications for the unconditional moments of returns. For instance, if we call $\nu_0 = E(\nu_{0t}), \lambda_j = E(\lambda_{jt})$ and $\pi_j = E(\pi_{jt})$ $(j = 1, \ldots, k)$, the assumption of constant risk prices implies that $E(R_{f_j t}) - \nu_0 = \pi_j = \tau_j \lambda_j$, that $V(R_{f_j t}) = V(\nu_{0t}) + \lambda_j + \tau_j^2 V(\lambda_{jt})$, and that in general the returns on the basis portfolios will be serially correlated as long as $\tau_j \neq 0$ (see Fiorentini and Sentana, 1996). Similarly, the assumption of constant betas implies that the unconditional covariance matrix of the *innovations* in returns has an orthogonal k-factor structure. However, this is not generally the case for the unconditional covariance matrix of *gross* returns. In particular, the unconditional covariance matrix of $\mathbf{R}_{Nt}$ will be

$$\mathbf{\Sigma}_N = (\ell_N | \mathbf{B}_N) \begin{pmatrix} V(\nu_{0t}) & cov'(\nu_{0t}, \mathbf{\Lambda}_t \boldsymbol{\tau}) \\ & \mathbf{\Phi} \end{pmatrix} \begin{pmatrix} \ell'_N \\ \mathbf{B}'_N \end{pmatrix} + \quad {}_N$$

where $\mathbf{\Phi} = \mathbf{\Lambda} + V(\mathbf{\Lambda}_t \boldsymbol{\tau}), \mathbf{\Lambda} = V(\mathbf{f}_t) = E(\mathbf{\Lambda}_t)$ and $\quad {}_N = V(\mathbf{v}_{Nt}) = E(\quad {}_{Nt})$. Obviously, if in the zero-beta version of our model we assume that ν_{0t} is constant, the k factor structure will be preserved since $\mathbf{\Sigma}_N = \mathbf{B}_N \mathbf{\Phi} \mathbf{B}'_N + \quad {}_N$,. Furthermore, if the univariate processes for the factors are "strong" GARCH in the sense of Drost and Nijman (1993), the "factors" $\mathbf{f}_t + (\mathbf{\Lambda}_t - \mathbf{\Lambda})\boldsymbol{\tau}$ will be unconditionally uncorrelated. The same orthogonal k factor structure will be obtained for *excess* returns when a riskless portfolio exists.

Finally, if we call $\mu_i = E(\mu_{it})$ the (temporal) average "risk premium", the assumptions of constant betas and risk prices imply that our linear factor pricing model also holds on average, i.e.

$$\mu_i = \beta_{i1} \lambda_1 \tau_1 + \ldots + \beta_{ik} \lambda_k \tau_k \qquad [9]$$

2.3. Mean-variance implications

To illustrate the implications of our theoretical asset pricing restrictions on the conditional mean variance frontier, we shall consider a very simple version of our model, with a single common factor (whose unconditional variance is 1), and scalar idiosyncratic covariance matrix $\omega_t \mathbf{I}_N$, with $\omega_t > 0$, and $E(\omega_t) = \omega < \infty$. In this case, the vector of conditional expected returns for the N primitive assets and the conditional covariance matrix will be given by $\boldsymbol{\nu}_{Nt} = \nu_{0t}\ell_N + \boldsymbol{\mu}_{Nt} = \nu_{0t}\ell_N + \mathbf{b}_N\lambda_t\tau$ and $\boldsymbol{\Sigma}_{Nt} = \mathbf{b}_N\mathbf{b}'_N\lambda_t + \omega_t\mathbf{I}_N$.

It is convenient to write the conditional MV parabola in terms of ν_{0t} as $[\nu_{0t} + \bar{\mu}_t; (\text{C}_{Nt}/\text{D}_{Nt})\bar{\mu}_t^2 - 2(\tilde{\text{A}}_{Nt}/\text{D}_{Nt})\bar{\mu}_t + (\tilde{\text{B}}_{Nt}/\text{D}_{Nt})]$, with $\tilde{\text{A}}_{Nt} = \ell'_N\boldsymbol{\Sigma}_{Nt}^{-1}\boldsymbol{\mu}_{Nt}$ and $\tilde{\text{B}}_{Nt} = \boldsymbol{\mu}'_{Nt}\boldsymbol{\Sigma}_{Nt}^{-1}\boldsymbol{\mu}_{Nt}$. Two points are worth highlighting: the minimum conditional variance portfolio $[\nu_{0t} + (\tilde{\text{A}}_{Nt}/\text{C}_{Nt});$ $(1/\text{C}_{Nt})]$, and the maximum conditional Sharpe ratio portfolio $[\nu_{0t}$ $+(\tilde{\text{B}}_{Nt}/\tilde{\text{A}}_{Nt}); (\tilde{\text{B}}_{Nt}/\tilde{\text{A}}_{Nt}^2)]$, where we define
$[E(R_{pt} \mid I_{t-1}) - \nu_{0t}]/V^{1/2}(R_{pt} \mid I_{t-1})$
as the conditional Sharpe ratio of a portfolio.

Let $\bar{b}_N$ be the sample mean of the factor loadings for the N assets at hand, and $\bar{\sigma}_{Nb}^2$ their sample variance (with denominator N). Using the Woodbury formula, we get $\tilde{\text{A}}_{Nt} = \tau\lambda_t\bar{b}_{Nt}/[(\bar{\sigma}_{Nb}^2 + \bar{b}_N^2)\lambda_t + \omega_t/N]$,
$\tilde{\text{B}}_{Nt} = \tau^2\lambda_t^2(\bar{\sigma}_{Nb}^2 + \bar{b}_{Nt})/[(\bar{\sigma}_{Nb}^2 + \bar{b}_N^2)\lambda_t + \omega_t/N]$ and
$\text{C}_{Nt} = (\lambda_t N\bar{\sigma}_{Nt}^2 + \omega_t)/[(\bar{\sigma}_{Nb}^2 + \bar{b}_N^2)\lambda_t + \omega_t/N]$.

As we have seen, an important question is whether a conditionally riskless portfolio can be achieved as $N \to \infty$. Since the minimum variance attainable is $1/\text{C}_{Nt}$, a limiting riskless portfolio will exist unless $N\bar{\sigma}_{Nb}^2$ remains bounded in the limit. Such a situation trivially arises in our model if all but a finite set of factor loadings are equal, and also as a limiting case if $b_i = \beta + \varepsilon_i$, with $\varepsilon_i \sim iid(0, \omega^2/N)$. Importantly, if a limiting riskless portfolio exists, its expected return is ν_{0t}, and the efficient portfolios lie in a straight line with slope $\tau\lambda_t^{1/2}$ in conditional mean-standard deviation space. In that case, the higher the volatility, the higher the conditional Sharpe ratio attainable.

Let Q_{Nt} be the set of payoffs from all possible portfolios of the N primitive assets *with fixed weights* and finite unconditional second moments. Obviously, $Q_{Nt} \subset P_{Nt}$, but in addition to portfolios with unbounded unconditional second moments, we are excluding the returns of dynamic portfolio strategies based on information in I_{t-1}. From an empirical point of view, it is interesting to look at the corresponding unconditional MV frontier, which we define here as the collection of portfolios in Q_{Nt} whose weights are the solution to the related program $\min_w \mathbf{w}'_N \mathbf{\Sigma}_N \mathbf{w}_N$ subject to $\mathbf{w}'_N \ell_N = 1$ and $\mathbf{w}'_N \boldsymbol{\nu}_N = \bar{\nu}$ for $\bar{\nu} \in \mathbb{R}$, where $\boldsymbol{\nu}_N = E(\mathbf{R}_{Nt}) = E(\boldsymbol{\nu}_{Nt})$ by the law of iterated expectations, and $\mathbf{\Sigma}_N = V(R_{Nt}) = E(\mathbf{\Sigma}_{Nt}) + V(\boldsymbol{\nu}_{Nt})$. Not surprisingly, in unconditional MV space, such unconditionally frontier portfolios lie on the parabola $[\nu; (\text{C}_N/\text{D}_N)\nu^2 - 2(\text{A}_N/\text{D}_N)\nu + (\text{B}_N/\text{D}_N)]$, where $\text{A}_N = \ell'_N \mathbf{\Sigma}_N^{-1} \boldsymbol{\nu}_N$, $\text{B}_N = \boldsymbol{\nu}'_N \mathbf{\Sigma}_N^{-1} \boldsymbol{\nu}_N$, $\text{C}_N = \ell'_N \mathbf{\Sigma}_N^{-1} \ell_N$ and $\text{D}_N = \text{B}_N \text{C}_N - \text{A}_N^2$.

If in the zero-beta version of the example above we make the extra assumption that $\nu_{0t} = \nu_0 \; \forall t, \mathbf{\Sigma}_N = \mathbf{b}_N \mathbf{b}'_N \phi + \omega \mathbf{I}_N$, where $\phi = 1 + \tau^2 V(\lambda_t)$, and the parabola can be written as $[\nu_0 + \mu; (\text{C}_N/\text{D}_N)\mu^2 - 2(\tilde{\text{A}}_N/\text{D}_N)\mu + (\tilde{\text{B}}_N/\text{D}_N)]$, where $\tilde{\text{A}}_N = \ell'_N \mathbf{\Sigma}_N^{-1} \boldsymbol{\mu}_N$, $\tilde{\text{B}}_N = \boldsymbol{\mu}'_N \mathbf{\Sigma}_N^{-1} \boldsymbol{\mu}_N$, with $\boldsymbol{\mu}_N = \nu_N - \nu_0 \ell_N$. It can then be shown that any portfolio which is on the unconditional MV frontier of Q_{Nt}, is also a conditional frontier portfolio for P_{Nt}.[5] This result is different from the one in Hansen and Richard (1987), who show that any portfolio which is on the unconditional MV frontier for P_{Nt} (not Q_{Nt}) must also be on its conditional mean-variance frontier, but not vice versa.[6] The unconditional mean-variance frontier for P_{Nt} can be generated as $R^*_{Nt} + \gamma p^{\#}_{Nt}$, where $\gamma = [\bar{\nu} - E(R^*_{Nt})]/E(p^{\#}_{Nt})$ (see Hansen and Richard, 1987), and it is closer to the variance axis than the unconditional mean-variance frontier for Q_{Nt}.

When a riskless asset is available, though, it is customary to consider an unconditional mean variance frontier for excess returns, $\mathbf{r}_{Nt} = \mathbf{R}_{Nt} - R_{0t}\ell_N$, as the solution to the program $\min_w \mathbf{w}'_N \mathbf{\Sigma}_N \mathbf{w}_N$ subject to $\mathbf{w}'_N \ell_N = 1$ and $\mathbf{w}'_N \boldsymbol{\mu}_N = \bar{\mu}$, where $\boldsymbol{\mu}_N = E(\mathbf{r}_{Nt})$ and $\bar{\Sigma}_N = V(\mathbf{r}_{Nt})$ (cf. Gibbons, Ross and Shanken, 1989). In mean-variance space for excess returns, we can write the frontier as $[\mu; (\bar{\text{C}}_N/\bar{\text{D}}_N)\mu^2 -$

[5]The proof is based on the fact that for such a restricted model, $w^h_{Nt} = w^h_N/\lambda_t$ and $w^g_{Nt} + \nu_0 w^h_{Nt} = w^g_N + \nu_0 w^h_N$ (cf. footnote (3)).

[6]For instance, R^+_{Nt} is a conditionally efficient portfolio for P_{Nt}, but it is not unconditionally efficient unless $C(p^+_{Nt}) = \text{A}_{Nt}/(1+\text{B}_{Nt})$ is constant.

$2(\bar{A}_N/\bar{D}_N)\mu + (\bar{B}_N/\bar{D}_N)]$, where $\bar{A}_N = \ell'_N \boldsymbol{\Sigma}_N^{-1} \boldsymbol{\mu}_N$, $\bar{B}_N = \boldsymbol{\mu}'_N \boldsymbol{\Sigma}_N^{-1} \boldsymbol{\mu}_N$, $\bar{C}_N = \ell'_N \boldsymbol{\Sigma}_N^{-1} \ell_N$ and $\bar{D}_N = \bar{B}_N \bar{C}_N - \bar{A}_N^2$. Two points are again worth mentioning: the minimum unconditional variance portfolio $[(\bar{A}_N/\bar{C}_N); (1/\bar{C}_N)]$, and the maximum unconditional Sharpe ratio portfolio $[(\bar{B}_N/\bar{A}_N); (\bar{B}_N/\bar{A}_N^2)]$, where we define the unconditional Sharpe ratio of a portfolio as $E(r_{pt})/V^{1/2}(r_{pt})$.

Again, it can be shown that any portfolio which is on this unconditional mean-variance frontier is also on all the conditional mean-variance frontiers for P_{Nt}. As a result, some of the implications of the model can be validly tested in terms of unconditional moments of excess returns.

3. Econometric methodology

3.1. Observed limiting factor representing portfolios

Under the assumption of conditional normality, the model can be estimated for N assets simultaneously by maximum likelihood. However, estimation can be considerably simplified if we prespecify the k limiting basis portfolios, $R_{f_1t}, \ldots, R_{f_kt}$.

With a riskless asset

If we add the k diversified basis portfolios to the list of N assets under consideration, the $N + k$ equations for excess returns are:

$$\begin{aligned}
r_{1t} &= \beta_{11}(\tau_1\lambda_{1t} + f_{1t}) + \ldots + \beta_{1k}(\tau_k\lambda_{kt} + f_{kt}) + v_{1t} \\
&\vdots \\
r_{Nt} &= \beta_{N1}(\tau_1\lambda_{1t} + f_{1t}) + \ldots + \beta_{Nk}(\tau_k\lambda_{kt} + f_{kt}) + v_{Nt} \\
r_{f1t} &= (\tau_1\lambda_{1t} + f_{1t}) \\
&\vdots \\
r_{fkt} &= (\tau_k\lambda_{kt} + f_{kt})
\end{aligned}$$

or in matrix notation

$$\begin{aligned}
\mathbf{r}_{Nt} &= \mathbf{B}_N(\boldsymbol{\Lambda}_t\boldsymbol{\tau} + \mathbf{f}_t) + \mathbf{v}_{Nt} = \mathbf{B}_N\mathbf{r}_{ft} + \mathbf{v}_{Nt} \\
\mathbf{r}_{ft} &= (\boldsymbol{\Lambda}_t\boldsymbol{\tau} + \mathbf{f}_t)
\end{aligned} \quad [10]$$

The joint likelihood function of $\mathbf{r}_{Nt}, \mathbf{r}_{ft}$ (conditional on I_{t-1}) can be factorized into the marginal of $\mathbf{r}_{ft}$ (given I_{t-1}) times the conditional of $\mathbf{r}_{Nt}$ given $\mathbf{r}_{ft}$ (and I_{t-1}). Conditional on the past, the marginal of $\mathbf{r}_{ft}$ has mean vector $\mathbf{\Lambda}_t\boldsymbol{\tau}$ and diagonal covariance matrix $\mathbf{\Lambda}_t$, whereas the conditional of $\mathbf{r}_{Nt}$ has mean $\mathbf{B}_N\mathbf{r}_{ft}$, and diagonal covariance matrix $_{Nt}$. The log-likelihood function for $\mathbf{r}_{Nt}, \mathbf{r}_{ft}$ can thus be written (apart from constants) as:

$$-\frac{1}{2}\sum_{t=1}^{T}\left\{\ln|\ \ _{Nt}| + tr\left[\ \ ^{-1}_{Nt}(\mathbf{r}_{Nt} - \mathbf{B}_N\mathbf{r}_{ft})(\mathbf{r}_{Nt} - \mathbf{B}_N\mathbf{r}_{ft})'\right]\right\}$$
$$-\frac{1}{2}\sum_{t=1}^{T}\left\{\ln|\mathbf{\Lambda}_t| + tr\left[\mathbf{\Lambda}_t^{-1}(\mathbf{r}_{ft} - \mathbf{\Lambda}_t\boldsymbol{\tau})(\mathbf{r}_{ft} - \mathbf{\Lambda}_t\boldsymbol{\tau})'\right]\right\} \qquad [11]$$

Given our parametrisation of conditional variances as *univariate* GQARCH processes, such a factorization performs a sequential cut on the joint log-likelihood function which makes $\mathbf{r}_{ft}$ weakly and strongly exogenous for $\mathbf{B}_N$ and the parameters in $_{Nt}$ (see Engle, Hendry and Richard, 1983). As a result, if we further assume that $\omega_{iit} = \omega_{ii}\ \forall t$, the ML estimates of $_N$ and $\mathbf{B}_N$ will be $\hat{\ }_N = dg(\tilde{\ }_N)$, where:

$$\tilde{\ }_N = \frac{1}{T}\sum_{t=1}^{T}\hat{\mathbf{v}}_{Nt}\hat{\mathbf{v}}'_{Nt} = \frac{1}{T}\sum_{t=1}^{T}(\mathbf{r}_{Nt}\mathbf{r}'_{Nt} - \hat{\mathbf{B}}_N\mathbf{r}_{ft}\mathbf{r}'_{Nt}) \qquad [12]$$

is the sample covariance matrix of the OLS residuals $\hat{\mathbf{v}}_{Nt} = \mathbf{r}_{Nt} - \hat{\mathbf{B}}_N\mathbf{r}_{ft}$, and:

$$\hat{\mathbf{B}}_N = (\sum_{t=1}^{T}\mathbf{r}_{Nt}\mathbf{r}_{Nt})(\sum_{t=1}^{T}\mathbf{r}_{ft}\mathbf{r}'_{ft})^{-1} \qquad [13]$$

Note that the i^{th} row of $\hat{\mathbf{B}}_N$ contains the coefficient estimates in the univariate OLS regression of each r_{it} on $\mathbf{r}_{ft}$. Such estimators remain consistent (albeit inefficient) when ω_{iit} follow univariate GQARCH processes instead, and furthermore, heteroskedasticity-robust standard errors will not be required as the regressors are strongly exogenous. Also note that there is no efficiency loss in estimating the model only for the N assets of interest.

The value of the conditional component of the log-likelihood function at the optimum will be $-\frac{T}{2}\{N \ln 2\pi + \ln |dg(\tilde{\ }_N)| + N\}$, with $\ln |dg(\tilde{\ }_N)| = \sum_{i=1}^{N} \ln \tilde{\omega}_{ii}$

Similarly, since $\mathbf{\Lambda}_t$ is diagonal, the second part of expression [11] can be written as:

$$-\frac{1}{2}\sum_{j=1}^{k}\sum_{t=1}^{T}\left[\ln |\lambda_{jt}| + (r_{f_j t} - \tau_j \lambda_{jt})^2/\lambda_{jt}\right]$$

and the ML estimates of $\boldsymbol{\tau}$ and the parameters in $\mathbf{\Lambda}_t$ can be obtained from k univariate GQARCH in mean models for $r_{f_j t}$ $(j = 1, \ldots, k)$, but do not have closed form expressions unless λ_{jt} is assumed constant. In such a special case, if we call $\hat{\pi}_j$ the sample mean of $r_{f_j t}$ (i.e. $T^{-1}\sum_{t=1}^{T} r_{f_j t}$), and $\hat{\lambda}_j = T^{-1}\sum_{t=1}^{T}(r_{f_j t} - \hat{\pi}_j)^2$ its sample variance, we find that $\hat{\tau}_j = \hat{\pi}_j/\hat{\lambda}_j$, and the value at the optimum of the component of the log-likelihood corresponding to $r_{f_j t}$ is $-T/2\{\ln 2\pi + \ln \hat{\lambda}_j + 1\}$. Note that $\hat{\pi}_j$ remains consistent when λ_{jt} is time-varying, but this is not the case for $\hat{\lambda}_j$ or $\hat{\tau}_j$, whose *plims* are $\lambda_j + \tau_j^2 V(\lambda_{jt})$ and $\tau_j(1 + \tau_j^2 V(\lambda_{jt})/\lambda_j)^{-1}$ respectively. Nevertheless, tests of $H_0 : \pi_j = 0$ with the correct asymptotic size can be computed on the basis of the usual t-ratio because the serial correlation in $r_{f_j t}$ disappears under the null.

But if the asset pricing restrictions implicit in our model do not hold because average expected returns on the assets are unrestricted, we have:

$$\begin{aligned}
\mathbf{r}_{ft} &= \boldsymbol{\alpha}_f + (\mathbf{\Lambda}_t\boldsymbol{\tau} + \mathbf{f}_t) \\
\mathbf{r}_{Nt} &= \boldsymbol{\alpha}_N + \mathbf{B}_N(\mathbf{\Lambda}_t\boldsymbol{\tau} + \mathbf{f}_t) + \mathbf{v}_{Nt} = \boldsymbol{\alpha}_N^* + \mathbf{B}_N\mathbf{r}_{ft} + \mathbf{v}_{Nt}
\end{aligned}$$

where $[\boldsymbol{\alpha}_N]_i = \alpha_i$ is Jensen's alpha for asset i, and $\boldsymbol{\alpha}_N^* = \boldsymbol{\alpha}_N - \mathbf{B}_N\boldsymbol{\alpha}_f$.

We can again factorize the joint likelihood function into the marginal component for $\mathbf{r}_{ft}$, and the conditional component for $\mathbf{r}_{Nt}$ given $\mathbf{r}_{ft}$. Thus, we can efficiently estimate $\boldsymbol{\alpha}_f, \boldsymbol{\tau}$ and the parameters in $\mathbf{\Lambda}_t$ by means of k univariate GQARCH-M models for $r_{f_j t}$ $(j = 1, \ldots, k)$ with a constant in the mean, whereas the ML estimates of $\boldsymbol{\alpha}_N^*, \mathbf{B}_N$ and $_N$ can be obtained from N univariate OLS regressions of r_{it} on a *constant* and $\mathbf{r}_{ft}$. If we call $\bar{\ }_N$ the sample covariance matrix of the OLS residuals (with denominator T), the value of the conditional log-likelihood function at the optimum is $-T/2\{N \ln 2\pi + \ln |dg(\bar{\ }_N)| + N\}$, with $\ln |dg(\bar{\ }_N)| = \sum_{i=1}^{N} \ln \bar{\omega}_{ii}$.

Hence, the contribution of the conditional distribution to the likelihood ratio (LR) test of the hypothesis of unbiased pricing, will be $T(\ln|dg(\tilde{\ }_N)| - \ln|dg(\bar{\ }_N)|) = T\sum_{i=1}^{N}(\ln\tilde{\omega}_{ii} - \ln\bar{\omega}_{ii})$, which under the null, follows a χ^2 distribution with N degrees of freedom. The remaining contributions (up to k) come from the individual likelihood ratio tests of zero α_{f_j} associated with those basis portfolios with time-varying conditional variances; for if λ_{jt} is constant, α_{fj} cannot be separately identified, and the value of the marginal likelihood function at the optimum is unchanged.

We can relax the assumptions of diagonality and constancy for $\ _N$ separately, and still retain substantial computational gains. If we allow for a non-diagonal but constant idiosyncratic covariance matrix, (i.e. $\omega_{ij} \neq 0$), the analysis above goes through unchanged except for the fact that the $dg()$ operator no longer appears in the log-likelihood expressions. In fact, the test statistic $T(\ln|\tilde{\ }_N| - \ln|\bar{\ }_N|)$ is the LR version of the Gibbons, Ross and Shanken (1987) test for the mean-variance efficiency of a portfolio of k other portfolios. If $\ _{Nt}$ is really constant, their F version has an exact finite sample distribution under our assumptions.

On the other hand, if $\ _{Nt}$ is diagonal, with typical element given by a univariate GQARCH model, efficient estimates of the factor loadings and the conditional variance parameters of the idiosyncratic terms are obtained from univariate GQARCH regressions of each asset excess returns on the excess returns of the basis portfolios. Note that the assumption of an exact factor structure implies that there is no efficiency gains in using system estimation techniques.

Without a riskless asset

Let's assume initially that $\nu_{0t} = \nu_0\ \forall t$. The zero-beta version of our model can be expressed as:

$$\begin{aligned}
\mathbf{R}_{ft} &= \nu_0\ell_k + \quad (\mathbf{\Lambda}_t\boldsymbol{\tau} + \mathbf{f}_t) \\
\mathbf{R}_{Nt} &= \nu_0\ell_N + \quad \mathbf{B}_N(\mathbf{\Lambda}_t\boldsymbol{\tau} + \mathbf{f}_t) \quad + \mathbf{v}_{Nt} \\
&= \nu_0\ell_N + \quad \mathbf{B}_N(\mathbf{R}_{ft} - \nu_0\ell_k) \quad + \mathbf{v}_{Nt}
\end{aligned}$$

For a given value of ν_0, the maximum likelihood estimates of all the other parameters can be easily obtained as follows. Let $r_{it}(\nu_0) =$

$R_{it} - \nu_0$, for $i = 1, 2, \ldots, N$ and $r_{f_j t}(\nu_0) = R_{f_j t} - \nu_0$ for $j = 1, \ldots, k$. Since this is a variable transformation with a Jacobian equal to the identity matrix, the value of the likelihood function is unaffected. The marginal distribution of $\mathbf{r}_{ft}(\nu_0)$ given I_{t-1} has mean $\mathbf{\Lambda}_t \boldsymbol{\tau}$ and diagonal covariance matrix $\mathbf{\Lambda}_t$, whereas the conditional distribution of $\mathbf{r}_{Nt}(\nu_0)$ given $\mathbf{r}_{ft}(\nu_0)$ (and I_{t-1}) has mean $\mathbf{B}_N \mathbf{r}_{ft}(\nu_0)$ and covariance matrix $\ _{Nt}$. The estimation procedures set up in the previous section can then be straightforwardly applied to obtain estimates of all other parameters as a function of ν_0. On this basis, the ML estimate of $\nu_0, \hat{\nu}_0$, can be obtained by means of a grid search. The computational advantages of such a procedure come from the fact that the original $(N+k)$-variate estimation problem has thus been reduced to $(N+k)$ univariate ones, which are much simpler to solve. This is particularly true when there are closed form solutions for some of the univariate estimation problems.

As a simple example, consider the situation in which $\ _{Nt}$ is assumed time-invariant, but not necessarily diagonal. Then, the i^{th} row of $\hat{\mathbf{B}}_N(\nu_0)$ can be obtained from the OLS regression of $r_{it}(\nu_0)$ on $\mathbf{r}_{ft}(\nu_0)$, whereas $\hat{\ }_N(\nu_0)$ will be the sample covariance matrix of the regression residuals $\hat{v}_{it}(\nu_0)$, with denominator T. Therefore, the value of the concentrated conditional component of the log-likelihood function will be $-T/2\{N \ln 2\pi + \ln|\hat{\ }(\nu_0)| + N\}$. If in addition the factor representing portfolios have constant conditional means and variances, so that the value of ν_0 does not really affect the marginal component of the log-likelihood function, it turns out that $\hat{\nu}_0$ can be obtained by solving a simple quadratic equation (see Shanken, 1985, for details).

Under the alternative hypothesis that average expected returns on the assets are unrestricted, estimation is even simpler as ν_0 is not separately identified, and the model becomes:

$$\begin{array}{rllll} \mathbf{R}_{ft} = & \boldsymbol{\alpha}_f + & \nu_0 \ell_k + & (\mathbf{\Lambda}_t \boldsymbol{\tau} + \mathbf{f}_t) & \\ \mathbf{R}_{Nt} = & \boldsymbol{\alpha}_N + & \nu_0 \ell_N + & \mathbf{B}_N(\mathbf{\Lambda}_t \boldsymbol{\tau} + \mathbf{f}_t) & + \mathbf{v}_{Nt} \\ = & \boldsymbol{\alpha}_N^* + & & \mathbf{B}_N \mathbf{R}_{ft} & + \mathbf{v}_{Nt} \end{array}$$

with $\boldsymbol{\alpha}_N^* = \boldsymbol{\alpha}_N + \nu_0 \ell_N - \mathbf{B}_N \boldsymbol{\alpha}_f^*$, and $\boldsymbol{\alpha}_f^* = \boldsymbol{\alpha}_f + \nu_0 \ell_k$.

Again, we can factorize the joint likelihood function into the marginal of $\mathbf{R}_{ft}$, and the conditional of $\mathbf{R}_{Nt}$ given $\mathbf{R}_{ft}$. The only difference with respect to the riskless asset case is that the χ^2 distributions for the likelihood ratio tests have now one degree of freedom less, as under the null, we have one parameter more, namely ν_0.

If ν_{0t} is not constant, we can still concentrate the log-likelihood function by working in terms of $r_{it}(\nu_{0t})$, but now we will be left with one incidental parameter per time period (i.e. $\nu_{01}, \ldots, \nu_{0T}$). Nevertheless, ν_{0t} behaves like a time dummy in panel data regressions, which suggests the following procedure. Let $R_{wt} = \sum_{i=1}^{N} w_i R_{it} = \mathbf{w}'_N \mathbf{R}_{Nt}$ (with $\mathbf{w}'_N \ell_N = 1$) be the return on a portfolio of the N assets with fixed weights. For instance, if $w_N = N^{-1}\ell_N$, we obtain the (cross-sectional) mean return for period $t, \bar{R}_{Nt}$. Given our assumptions (cf. [8]),

$$\begin{aligned} R_{wt} = {} & \nu_{0t} + \beta_{w1}(\lambda_{1t}\tau_1 + f_{1t}) + \ldots + \beta_{wk}(\lambda_{kt}\tau_k + f_{kt}) + v_{wt} \\ = {} & \nu_{0t} + \boldsymbol{\beta}'_w(\boldsymbol{\Lambda}_t \tau + \mathbf{f}_t) + v_{wt} \end{aligned}$$

where $\boldsymbol{\beta}_w = \mathbf{w}'_N \mathbf{B}_N$ and $v_{wt} = \mathbf{w}'_N \mathbf{v}_{Nt}$. Then, if we subtract R_{wt} from the N assets under consideration and the k basis portfolios, we get

$$\begin{aligned} \mathbf{R}_{ft} - R_{wt}\ell_k &= (\mathbf{I}_k - \ell_k \boldsymbol{\beta}'_w)(\boldsymbol{\Lambda}_t \boldsymbol{\tau} + \mathbf{f}_t) - v_{wt}\ell_k \\ \mathbf{R}_{Nt} - R_{wt}\ell_N &= (\mathbf{B}_N - \ell_N \boldsymbol{\beta}'_w)(\boldsymbol{\Lambda}_t \boldsymbol{\tau} + \mathbf{f}_t) + \mathbf{v}_{Nt} - v_{wt}\ell_N \end{aligned}$$

Therefore, the basic asset pricing restrictions that the coefficients on f_{jt} and λ_{jt} are proportional, with the same factor of proportionality for all assets, still hold for $r_{it}(R_{wt}) = R_{it} - R_{wt}$. However, the factor loadings should be interpreted as measuring the differential effect of a given factor on the return of each asset and the return on R_{wt}.

If R_{wt} were well diversified (i.e. $v_{wt} = 0$), the exact factor structure would be maintained, and $\mathbf{r}_{ft}(R_{wt})$ will be an exact time-invariant transformation of $\mathbf{r}_{ft}(\nu_{0t})$. In particular, $\mathbf{r}_{ft}(R_{wt}) = (\mathbf{I}_k - \ell_k \boldsymbol{\beta}'_w)\mathbf{r}_{ft}(\nu_{0t})$. Then, provided that $(\mathbf{I}_k - \ell_k \boldsymbol{\beta}'_w)$ has full rank (which is true unless $\boldsymbol{\beta}'_w \ell_k = 1$), we can apply the method described in Section 3.3 below to estimate the relevant parameters. For instance, if we assume constant idiosyncratic variances, the estimation of the transformed parameters $\mathbf{B}^*_N = (\mathbf{B}_N - \ell_N \boldsymbol{\beta}'_w)(\mathbf{I}_k - \ell_k \boldsymbol{\beta}'_w)^{-1}$ will entail running N OLS regressions of each $r_{it}(R_{wt})$ on $\mathbf{r}_{ft}(R_{wt})$. Importantly, if we add a constant to such regressions, the associated coefficient should still be zero under the null of correct pricing, but its interpretation will change, as it will now measure asset $i's$ mispricing relative to average mispricing.

In practice, v_{wt} is unlikely to be 0 for finite N. As a result, the exact factor structure is lost since $cov_{t-1}(v_{it}-v_{wt}, v_{lt}-v_{wt}) = \mathbf{w}'_N \quad {}_N\mathbf{w}_N - w_i\omega_{iit} - w_l\omega_{llt}$. More importantly, there will be an idiosyncratic component in $r_{f_jt}(R_{wt})$, which invalidates the estimation procedures above. Nevertheless, we can develop an asymptotic theory which justifies their use when N is large. As an example, let's take $R_{wt} = \bar{R}_{Nt}$ in a single factor model with constant covariance matrix. Let $\bar{\omega}_N$ be the average idiosyncratic variance. Then, $cov_{t-1}(v_{it}-v_{wt}, v_{lt}-v_{wt}) = N^{-1}(\bar{\omega}_N - \omega_{ii} - \omega_{ll})$, whereas the conditional regression coefficient of $r_{it}(\bar{R}_{Nt})$ on $r_{f_1t}(\bar{R}_{Nt})$ is $[(\beta_{i1} - \bar{b}_N)(1-\bar{b}_N) + N^{-1}(\bar{\omega}_N - \omega_{ii})]/[(1-\bar{b}_N)^2 + N^{-1}\bar{\omega}_N]$. As N increases, these terms converge to 0 and $(\beta_{i1} - \bar{b}_N)/(1-\bar{b}_N)$ respectively, and our estimation procedures will be valid.[7]

3.2. Unobserved limiting factor representing portfolios

The methods described in Section 3.1 suggest an iterative procedure to estimate the model when diversified factor representing portfolios are not observed.

With a riskless asset

The equations for the N available assets can be written as:

$$\mathbf{r}_{Nt} = \mathbf{B}_N\mathbf{\Lambda}_t\boldsymbol{\tau} + \mathbf{B}_N\mathbf{f}_t + \mathbf{v}_{Nt} \qquad [14]$$

Models like [14] are usually estimated for all N assets simultaneously by maximum likelihood under the assumption of joint conditional normality for $\mathbf{r}_{Nt}$. Ignoring initial conditions and constants, the log-

[7] If we knew $\quad_N$, the finite sample approximation could be improved by choosing the minimum idiosyncratic variance portfolio $\ell'_N \quad {}_N^{-1}\mathbf{R}_{Nt}/(\ell'_N \quad {}_N^{-1}\ell_N)$ as R_{wt}.

likelihood function takes the value

$$-\frac{1}{2}\sum_{t=1}^{T}\{\ln|\mathbf{B}_N\mathbf{\Lambda}_t\mathbf{B}'_N + \quad_{Nt}| \cdots$$

$$\cdots + tr[(\mathbf{B}_N\mathbf{\Lambda}_t\mathbf{B}'_N + \quad_{Nt})^{-1}\mathbf{v}_{Nt}\mathbf{v}'_{Nt}]\} \qquad [15]$$

where $\mathbf{v}_{Nt} = \mathbf{r}_{Nt} - \mathbf{B}_N\mathbf{\Lambda}_t\boldsymbol{\tau}$. Since the first order conditions are particularly complicated in this case, a numerical approach is usually required (see King, Sentana and Wadhwani, 1994). In order to avoid the non-measurability of conditional variances with respect to the econometrician's information set, which is smaller than that of the agents', we have adopted the correction in Harvey, Ruiz and Sentana (1992). For the GQARCH(1,1) case in particular, this can be achieved by an equation of the form:

$$\begin{aligned}
\lambda_{jt} = & \ \psi_{j0} + \psi_{j1}E(f_{jt-1}^2 \mid \mathbf{r}_{Nt-1}) + \gamma_{j1}E(f_{jt-1} \mid \mathbf{r}_{Nt-1}) + \delta_{j1}\lambda_{jt-1} \\
= & \ \psi_{j0} + \psi_{j1}[E^2(f_{jt-1} \mid \mathbf{r}_{Nt-1}) + V(f_{jt-1} \mid \mathbf{r}_{Nt-1})] \\
& + \gamma_{j1}E(f_{jt-1} \mid \mathbf{r}_{Nt-1}) + \delta_{j1}\lambda_{jt-1}
\end{aligned}$$

Notice that the measurability of λ_{jt} with respect to $\mathbf{r}_{Nt-1}$ is achieved by replacing the unobserved factors by their Kalman filter estimates, and including a correction in the standard ARCH terms which reflects the uncertainty in the factor estimates.

The EM algorithm provides a convenient way to obtain initial values as close to the optimum as desired (see Demos and Sentana, 1996). For clarity of exposition, we shall concentrate on the case of constant conditional variances. To solve the scale indeterminacy, we shall assume that $\mathbf{\Lambda} = \mathbf{I}_k$, which implies that $\boldsymbol{\tau} = \boldsymbol{\pi}$. Similarly, to differentiate common from idiosyncratic effects with a finite number of assets, we shall only consider exact factor structures.

Had we data on $\mathbf{r}_{ft}$, the method described in Section 3.1.1 above could be used to estimate all the model parameters. With the extra assumptions, the log-likelihood function for $\mathbf{r}_{Nt}, \mathbf{r}_{ft}$ in equation [11] simplifies to:

$$
\begin{aligned}
&-\frac{T}{2}\ln|\quad_N| + \frac{1}{2}\sum_{t=1}^{T} tr[\quad_N^{-1}(\mathbf{r}_{Nt} - \mathbf{B}_N r_{ft})(\mathbf{r}_{Nt} - \mathbf{B}_N r_{ft})'] \\
&-\frac{T}{2}\ln|\mathbf{I}_k| - \frac{1}{2}\sum_{j=1}^{k}\sum_{t=1}^{T}(r_{f_j t} - \pi_j)^2
\end{aligned}
$$

At each iteration, the EM algorithm maximizes the expected value of the above expression conditional on all the observed asset returns and the current parameter estimates.

Let $\mathbf{r}_{ft|T} = \boldsymbol{\pi} + \mathbf{B}'_N(\mathbf{B}_N\mathbf{B}'_N + \quad_N)^{-1}(\mathbf{r}_{Nt} - \mathbf{B}_N\boldsymbol{\pi})$ be the expected value of $\mathbf{r}_{ft}$ given $\mathbf{r}_{Nt}$, and $\boldsymbol{\Psi}_{t|T} = \mathbf{I}_k - \mathbf{B}'_N(\mathbf{B}_N\mathbf{B}'_N + \quad_N)^{-1}\mathbf{B}_N$ its associated mean square error, which can be evaluated via the Kalman filter. With this notation, the objective function at iteration $n+1$ becomes:

$$
\begin{aligned}
&-\frac{T}{2}\ln|\quad_N| \\
&-\frac{1}{2}\sum_{t=1}^{T} tr(\quad_N^{-1}[(\mathbf{r}_{Nt} - \mathbf{B}_N\mathbf{r}^{(n)}_{ft|T})(\mathbf{r}_{Nt} - \mathbf{B}_N\mathbf{r}^{(n)}_{ft|T})' + \mathbf{B}_N\boldsymbol{\Psi}^{(n)}_{t|T}\mathbf{B}'_N]) \\
&-\frac{T}{2}\ln|\mathbf{I}_k| - \frac{1}{2}\sum_{j=1}^{k}\sum_{t=1}^{T}[(r^{(n)}_{f_j t|T} - \pi_j)^2 + \Psi^{(n)}_{fjt|T}]\}
\end{aligned}
$$

where (n) refers to expressions evaluated at the current parameter estimates.

From the first order conditions we get:

$$
\begin{aligned}
\hat{\mathbf{B}}^{(n+1)}_N &= (\sum_{t=1}^{T}\mathbf{r}_{Nt}\mathbf{r}^{(n)\prime}_{ft|T})[\sum_{t=1}^{T}\mathbf{r}^{(n)}_{ft|T}\hat{\mathbf{r}}^{(n)\prime}_{ft|T} + \boldsymbol{\Psi}^{(n)}_{t|T}]^{-1} \\
\hat{\ }^{(n+1)}_N &= dg[T^{-1}\sum_{t=1}^{T}(\mathbf{r}_{Nt}\mathbf{r}_{Nt} - \hat{\mathbf{B}}^{(n+1)}_N\mathbf{r}^{(n)}_{ft|T}\mathbf{r}'_t)] \qquad [16] \\
\hat{\boldsymbol{\pi}}^{(n+1)} &= T^{-1}\sum_{t=1}^{T}\mathbf{r}^{(n)}_{ft|T}
\end{aligned}
$$

and one could then go on to show that the value of the log-likelihood function [15] at the optimum will be given by:

$$
\left[-\frac{T}{2}\{N\ln 2\pi + \ln\left|\hat{\mathbf{B}}_N\hat{\mathbf{B}}'_N + \hat{\ }_N\right| + N\}\right]
$$

where $\hat{\mathbf{B}}_N = \hat{\mathbf{B}}_N^{(\infty)}$ and $\hat{\ }_N = \hat{\ }_N^{(\infty)}$.

Notice the similarity between expressions [16] and the corresponding ones for case in which the basis portfolios are observed (cf. [12]-[13]). Here, the unobservable basis portfolios are replaced by their best (in the mean square error sense) estimates given the available assets. The main difference is that in computing the estimates of $\mathbf{B}_N$, we take into account the fact that $\mathbf{r}_{ft}$ is not really observed.

Again, if the asset pricing restrictions do not hold, we will have

$$\mathbf{r}_{Nt} = \boldsymbol{\alpha}_N + \mathbf{B}_N(\boldsymbol{\pi} + \mathbf{f}_t) + \mathbf{v}_{Nt} = \boldsymbol{\alpha}_N^* + \mathbf{B}_N\mathbf{f}_t + \mathbf{v}_{Nt}$$

The EM algorithm can still be applied, but now the "regressors" will include a constant. After tedious algebraic manipulations, one can prove that the value of the unrestricted log-likelihood function at the optimum will be given by $-\frac{T}{2}\{N \ln 2\pi + \ln |\bar{\mathbf{B}}_N \bar{\mathbf{B}}_N + \bar{\ }_N| + N\}$, where $\bar{\mathbf{B}}_N$ and $\bar{\ }_N$ are the unrestricted ML estimates of the model parameters. Hence, the likelihood ratio test will be $T(\ln \left|\hat{\mathbf{B}}_N \hat{\mathbf{B}}'_N + \hat{\ }_N\right| - \ln |\bar{\mathbf{B}}_N \bar{\mathbf{B}}'_N + \bar{\ }_N|)$, which is asymptotically distributed as a χ^2 variable with $N-k$ degrees of freedom under the null of unbiased pricing. The reduction in the number of degrees of freedom comes from the fact that under the null, we have k free parameters in $\boldsymbol{\pi}$ to explain all N means.

Demos and Sentana (1996) extend the EM algorithm to the case of time-varying conditional variances for the factors, but constant idiosyncratic variances. In this respect, they show that, irrespectively of the number of assets under consideration, the only difference that modelling the time-variation in the conditional variances of the factors makes is that at each EM iteration, one has to estimate k extra univariate dynamic heteroskedasticity in mean models. Their results confirm that the EM algorithm yields significant speed gains, and that it makes unnecessary the computation of good initial values. However, near the optimum it slows down significantly, and the best practical strategy then is to switch to a first derivative-based method.

Their estimation procedure could be modified to allow for time-varying idiosyncratic variances, but such an extension is unlikely to offer any computational advantages in models like [14] with no role for observable economic variables or weakly exogenous regressors (cf. the model in King, Sentana and Wadhwani, 1994, and Section 3.3 below).

Nevertheless, as we saw in Section 2.2, our assumptions imply that the unconditional covariance matrix of excess returns will be $\mathbf{\Sigma}_N = \mathbf{B}_N \mathbf{\Phi} \mathbf{B}'_N + \quad_N$, with $\mathbf{\Phi} = \mathbf{\Lambda} + V(\mathbf{\Lambda}_t \boldsymbol{\tau})$ diagonal, and that the (temporal) average risk premium is $\boldsymbol{\mu}_N = \mathbf{B}_N \mathbf{\Lambda} \boldsymbol{\tau} = \mathbf{B}_N \boldsymbol{\pi}$. Hence, if we estimate the model on the basis of [16] ignoring the time-variation in $\mathbf{\Lambda}_t$ and $\quad_{Nt}$, we obtain inconsistent estimates of $\boldsymbol{\tau}$, but consistent estimates of $\mathbf{B}_N$ and $\boldsymbol{\pi}$ (subject to the normalization $\mathbf{\Phi} = \mathbf{I}_k$) (cf. Section 3.1.1). Therefore, we can still test whether our asset pricing model is correct on average (over time) by assessing the significance of added constants on the basis of the likelihood ratio discussed above.

No riskless asset available

If $\nu_{0t} = \nu_0 \ \forall t$, and we knew its value, the EM algorithm explained in the previous section would provide a convenient way to obtain initial values as close to the optimum as desired for the constant variance case, by working in terms of $r_{1t}(\nu_0), \ldots, r_{Nt}(\nu_0)$. In general, we could carry out a grid search over ν_0, and for each value of this parameter maximize the likelihood function with respect to all the other unknown parameters. The value of ν_0 for which the maximum of the log-likelihood function conditional on this parameter is largest will be the ML estimator of our model. More formally, $\hat{\nu}_0 = \arg\min_{\nu_0} \ln \left| \hat{\mathbf{B}}_N(\nu_0) \hat{\mathbf{B}}'_N(\nu_0) + \hat{\ }_N(\nu_0) \right|$. However, this is a computationally inefficient procedure unless we can tightly bound the likely values of ν_0.

Under the alternative, expected returns on the assets are unrestricted, and the EM algorithm with a constant regressor discussed in the previous section can be applied to *gross* returns directly. The value of the likelihood ratio test will then be $T(\ln|\hat{\mathbf{B}}_N(\hat{\nu}_0)\hat{\mathbf{B}}'_N(\hat{\nu}_0) + \hat{\ }_N(\hat{\nu}_0)| - \ln|\bar{\mathbf{B}}_N \bar{\mathbf{B}}'_N + \bar{\ }_N|)$. Under the null, it is asymptotically distributed as a χ^2 with $N - k - 1$ degrees of freedom, since the asset pricing model uses now $k + 1$ free parameters for the mean.

If ν_{0t} is not assumed constant, we can again work in terms of $R_{it} - R_{wt}$.

If $v_{wt} = 0$, the procedures in Section 3.2.1 remain valid. Otherwise, they have to be interpreted as a large N approximation.

3.3. Partially observed factor representing portfolios

If we do not directly observe $\mathbf{R}_{ft}$, but instead have data on the excess returns on k diversified portfolios, $\mathbf{R}_{pt}$, which are a full-rank, time-invariant transformation of the limiting basis portfolios (i.e. $\mathbf{R}_{pt} = \mathbf{B}_p\mathbf{R}_{ft}$, with $\mid \mathbf{B}_p \mid \neq 0$), the procedures spelled out in Sections 3.1 and 3.2 can be advantageously combined (see the discussion at the end of Section 3.1.1, and Sentana, Shah and Wadhwani, 1995, for an empirical application in which $\mathbf{B}_p$ is lower triangular). For simplicity, we shall only consider the riskless asset case. The model for all available assets will be

$$\begin{aligned} \mathbf{r}_{pt} &= \mathbf{B}_p(\boldsymbol{\Lambda}_t\boldsymbol{\tau} + \mathbf{f}_t) & &= \mathbf{B}_p\mathbf{r}_{ft} \\ \mathbf{r}_{Nt} &= \mathbf{B}_N(\boldsymbol{\Lambda}_t\boldsymbol{\tau} + \mathbf{f}_t) & +\mathbf{v}_{Nt} &= \mathbf{B}_N\mathbf{r}_{ft} + \mathbf{v}_{Nt} \end{aligned}$$

Once more, the joint likelihood function of $\mathbf{r}_{Nt}, \mathbf{r}_{pt}$ (conditional on I_{t-1}) can be factorized into the marginal of $\mathbf{r}_{pt}$ (given I_{t-1}) times the conditional of $\mathbf{r}_{Nt}$ given $\mathbf{r}_{pt}$ (and I_{t-1}). Conditional on the past, the marginal distribution of $\mathbf{r}_{pt}$ corresponds to a k-variate k-factor GQARCH-M model (Engle, 1987) with mean vector $\mathbf{B}_p\boldsymbol{\Lambda}_t\boldsymbol{\tau}$ and covariance matrix $\mathbf{B}_p\boldsymbol{\Lambda}_t\mathbf{B}'_p$, whereas the conditional distribution of $\mathbf{r}_{Nt}$ has mean $\mathbf{B}^*_N\mathbf{r}_{pt}$, with $\mathbf{B}^*_N = \mathbf{B}_N\mathbf{B}_p^{-1}$, and diagonal covariance matrix ${}_{Nt}$.

The ML estimates of $\boldsymbol{\tau}$ and the parameters in $\mathbf{B}_p$ and $\boldsymbol{\Lambda}_t$ have to be obtained numerically on the basis of an expression analogous to [15] but with ${}_{pt} = \mathbf{0}$, whereas $\mathbf{B}^*_N$ and the parameters in ${}_{Nt}$ can be efficiently estimated using the procedures outlined in Section 3.1, because $\mathbf{r}_{pt}$ is weakly and strongly exogenous for them. Then, since the induced reparametrisation is one-to-one, by the invariance property of ML estimation, $\hat{\mathbf{B}}_N = \hat{\mathbf{B}}^*_N\hat{\mathbf{B}}_p$.

If we observe less than k basis portfolios, say h, the ML estimates of their conditional variance parameters and prices of risk can still be obtained by running h univariate GQARCH-M models. However, estimation of all the other parameters is not as straightforward as before. Let's call $\mathbf{r}'_{f_1t} = (r_{f_1t}, \ldots, r_{f_ht})'$ and partition all relevant vectors and matrices accordingly as $\mathbf{r}'_{ft} = (\mathbf{r}'_{f_1t}, \mathbf{r}'_{f_2t})$, $\mathbf{B}_N = (\mathbf{B}_{1N}, \mathbf{B}_{2N})$, $vecd'(\mathbf{\Lambda}_t) = [vecd'(\mathbf{\Lambda}_{1t}), vecd'(\mathbf{\Lambda}_{2t})]$ and $\boldsymbol{\tau}' = (\boldsymbol{\tau}'_1, \boldsymbol{\tau}'_2)$. The conditional distribution of $\mathbf{r}_{Nt}$ given $\mathbf{r}_{f_1t}$ has mean $\mathbf{B}_{1N}r_{f_1t} + \mathbf{B}_{2N}\mathbf{\Lambda}_{2t}\boldsymbol{\tau}_2$ and covariance matrix $\mathbf{B}_{2N}\mathbf{\Lambda}_{2t}\mathbf{B}'_{2N} + \;{}_{Nt}$. Hence, the procedures outlined in Section 3.2 can be used as if we had only $k - h$ common factors provided that we include $\mathbf{r}_{f_1t}$ as regressors.

A similar approach can still be used if we have data on $h < k$ portfolios which are a full-rank, time-invariant transformation of h basis portfolios. However, if the available h basis portfolios are a linear transformation of all k limiting factor representing portfolios, we can only add them to the list of N assets at hand, and estimate jointly using the procedures in Section 3.2. The only noticeable difference is that some idiosyncratic variances will be zero (see Sentana, 1994b). Such a situation arises when the market portfolio is a well diversified linear combination of all factors. More specifically,

$$r_{mt} = \beta_{m1}\lambda_{1t}\tau_1 + \ldots + \beta_{mk}\lambda_{kt}\tau_k + \beta_{m1}f_{1t} + \ldots + \beta_{mk}f_{kt}$$

The market beta for an asset will be given by

$$\beta_{imt} = \frac{cov(r_{it}, r_{mt} \mid I_{t-1})}{V(r_{mt} \mid I_{t-1})} = \frac{\beta_{m1}\beta_{i1}\lambda_{1t} + \ldots + \beta_{mk}\beta_{ik}\lambda_{kt}}{\beta_{m1}^2\lambda_{1t} + \ldots + \beta_{mk}^2\lambda_{kt}}$$

while the market price of risk will be

$$\tau_{mt} = \frac{E(r_{mt} \mid I_{t-1})}{V(r_{mt} \mid I_{t-1})} = \frac{\beta_{m1}\lambda_{1t}\tau_1 + \ldots + \beta_{mk}\lambda_{kt}\tau_k}{\beta_{m1}^2\lambda_{1t} + \ldots + \beta_{mk}^2\lambda_{kt}}$$

both of which are generally time-varying. A straightforward application of the CAPM yields $\mu_{it} = \beta_{imt}\mu_{mt}$, which for $k > 1$ is different from $\beta_{i1}\lambda_{1t}\tau_1 + \ldots + \beta_{ik}\lambda_{kt}\tau_k$, unless $\tau_{mt} = \tau_j/\beta_{mj}$ for all j and t. However, this difference is only due to the assumptions of constant betas and constant but unrestricted factor prices made for empirical tractability. At the theoretical level, if the market portfolio is diversified, the definition of τ_{jt} is such that the restriction $\tau_{jt} = \tau_{mt}\beta_{mjt}$ is always satisfied for $j = 1, \ldots, k$ (see Section 2.1 above). If we imposed the restriction $\tau_j = \tau_m\beta_{mj}\ \forall j$, we would estimate a conditional CAPM model with a constant market price of risk, but time-varying market betas, in which $\mu_{it} = \beta_{imt}\mu_{mt}$, and $\mu_{mt} = \tau_m\sigma_{mmt}$ (see Demos, Sentana and Shah, 1993).

Another interesting situation arises when we only observe (root-N) consistent estimates of the limiting factor representing portfolios. For simplicity, let's consider the case of constant idiosyncratic variances, in which we only observe the GLS factor representing portfolios based on the N assets at hand, $\mathbf{r}_{ft}^{GLS}$, where $\mathbf{r}_{ft}^{GLS} = (\mathbf{B}'_N \; {}_N^{-1}\mathbf{B}_N)^{-1}\mathbf{B}'_N \; {}_N^{-1}$ $\mathbf{r}_{Nt} = \mathbf{D}'_N\mathbf{r}_{Nt}$. These basis portfolios are different from the expected value of $\mathbf{r}_{ft}$ given $\mathbf{r}_{Nt}$, $\mathbf{r}_{ft|T}$, but closely related (see Sentana, 1994a for details).

Since knowledge of $\mathbf{r}_{ft}^{GLS}$ is tantamount to knowledge of $\mathbf{D}_N$, we can construct a known $N \times N$ matrix of full rank $\bar{\mathbf{D}}_N = (\mathbf{D}_N, \bar{\mathbf{D}}_{N_2})$ with $\bar{\mathbf{D}}_{N_2}$ arbitrary, so that the factor structure for $\mathbf{r}_{Nt}$ is preserved in the transformed observations $\bar{\mathbf{r}}'_{Nt} = \mathbf{r}'_{Nt}\bar{\mathbf{D}}_N = (\mathbf{r}'^{GLS}_{ft}, \bar{\mathbf{r}}_{N_2t})$, with $\bar{\mathbf{B}}'_N = \mathbf{B}'_N\bar{\mathbf{D}}_N = (\mathbf{I}_k, \bar{\mathbf{B}}'_{N_2})$ and $\bar{\;}_N = \bar{\mathbf{D}}'_N \; {}_N\bar{\mathbf{D}}_N$. It is then easy to see that $\mathbf{r}_{ft|T}$ and $\mathbf{\Psi}_{t|T}$ are not affected by the change of variables, and furthermore, that they can be computed from information on $\mathbf{r}_{ft}^{GLS}$ alone.

The joint distribution of $\bar{\mathbf{r}}_{Nt}$ given I_{t-1} can be factorized into the marginal of $\mathbf{r}_{ft}^{GLS}$, which is

$$N(\mathbf{\Lambda}_t\boldsymbol{\tau}, \mathbf{\Lambda}_t + \bar{\;}_{11}) \;\; \text{with} \;\; \bar{\;}_{11} = (\mathbf{B}'_N \; {}_N^{-1}\mathbf{B}_N)^{-1},$$

plus the conditional of $\bar{\mathbf{r}}_{N_2t}$ given $\mathbf{r}_{ft}^{GLS}$ (and I_{t-1}), which is

$$N(\bar{\mathbf{B}}_{N_2}\mathbf{r}_{ft}^{GLS}, \bar{\;}^*_{22}) \;\; \text{with} \;\; \bar{\;}^*_{22} = \bar{\;}_{22} - \bar{\mathbf{B}}_{N_2}\bar{\;}_{11}\bar{\mathbf{B}}'_{N_2}.$$

If ${}_N$ is unrestricted, this is a one-to-one reparametrisation for which the above factorization operates a sequential cut (see Engle, Hendry and Richard, 1983). As a result, efficient estimates of $\bar{\mathbf{B}}_{N_2}$ and $\bar{\;}^*_{22}$ can be obtained from $N-k$ OLS regressions of each $\bar{r}_{it}$ $(i = k+1, \ldots, N)$ on $\mathbf{r}_{ft}^{GLS}$. Estimates of $\boldsymbol{\tau}$, $\bar{\;}_{11}$ and the parameters in $\mathbf{\Lambda}_t$ would have to be obtained by numerically maximizing the joint log-likelihood function of $\mathbf{r}_{ft}^{GLS}$. Note that this function is not simply the sum of k univariate ones unless $(\mathbf{B}'_N \; {}_N^{-1}\mathbf{B}_N)$ is diagonal. Nevertheless, since the dependence of $\mathbf{\Lambda}_t$ on $\mathbf{B}_N$ and ${}_N$ is only through $\bar{\;}_{11}$, it turns out that the implied ML estimates of $\mathbf{B}_N$ are simply

OLS coefficient estimates in the regression of each r_{it} $(i = 1, \dots, N)$ on $\mathbf{r}_{ft}^{GLS}$.

4. Data

The database used contains arithmetic monthly returns (adjusted for dividends and stock splits) in percentage terms on 164 firms listed in the Spanish stock market between January 1963 and December 1992 (i.e. 360 observations) and a portfolio, hereinafter VW, which is a weighted average of all assets, with weights that depend on market capitalization at the end of the previous year. As a safe asset, we used T-bill returns on the secondary market after 1982, and the average lending rate from banks and saving institutions before (see Rubio, 1988, for details).

This is an unbalanced panel with different number of series in different periods. Although the methods described in Section 3 can be adapted accordingly, such modifications substantially increase the book-keeping costs of the computations, and may even be impractical. For instance, for a model with unobservable basis portfolios and constant conditional variances, we can first artificially balance the panel by setting to zero the missing observations. Then, we can still use expressions [16] except for a correction in the denominator of ${}_{N}^{(n+1)}$ to reflect the right number of observations for each series, provided that in computing $\hat{\mathbf{r}}_{ft}$ and Ψ_t we only use the assets with available observations in period t (see Demos and Sentana, 1992). A model with observations on the basis portfolios for the whole sample is simpler to handle, as long as we maintain the assumption of diagonal idiosyncratic variances. In such a situation, $\mathbf{B}_N$ and the parameters in ${}_{Nt}$ can still be estimated by means of univariate OLS regressions of each r_{it} on $\mathbf{r}_{ft}$ for the relevant sub-period, and even GQARCH regressions provided that there are no gaps, and the total number of observations is reasonable.

In Sentana (1995a) we avoided unbalanced panels by working with ten equally-weighted size-ranked portfolios constructed from all available assets in each period. Here, we initially balance the sample according to the "optimal" procedure set up in the appendix with two different objective functions, $\max T$ and $\max NT$. The first objective function trivially selects those shares that have been listed for the complete sample period, 48 in our case. The second one reduces the sample span to 309 periods, but adds another 10 firms which were uninterruptedly listed from January 1963 until at least September 1988, a mere increase of 642 observations (3.7%). Given the substantial degree of overlap between both datasets, and the fact that to capture the temporal variation in volatility at the monthly frequency what matters most is how large T is, we only use data on the returns of the 48 firms listed during the whole period.

In order to evaluate the effects of working with balanced datasets on estimation and testing, let's consider a situation with observed basis portfolios and constant but non-diagonal idiosyncratic covariance matrix, in which there are N_1 firms with observations for $t = 1, \ldots, T_1 < T$, and $N_2(= N - N_1)$ firms with observations for the whole sample (see Sentana, 1996, 1997). Our hypothesis of interest is that the asset pricing restrictions are correct. Let's partition $\mathbf{r}_{Nt}, \mathbf{B}_N$ and ${}_{N}$ conformably as

$$\mathbf{r}_{Nt} = \begin{pmatrix} \mathbf{r}_{N_1t} \\ \mathbf{r}_{N_2t} \end{pmatrix}, \; \mathbf{B}_N = \begin{pmatrix} \mathbf{B}_{N_1} \\ \mathbf{B}_{N_2} \end{pmatrix}, \quad {}_{N} = \begin{pmatrix} {}_{N_1N_1} & {}_{N_1N_2} \\ & {}_{N_1N_2} \end{pmatrix}$$

and for simplicity of notation, let's ignore dependence on the past.

For the last $T - T_1$ observations, the likelihood function can be factorized as the marginal distribution of $\mathbf{r}_{ft}$ times the distribution of $\mathbf{r}_{N_2t}$ given $\mathbf{r}_{ft}$. On the other hand, for the first T_1 observations, the likelihood function can be factorized as the marginal distribution of $\mathbf{r}_{ft}$, times the distribution of $\mathbf{r}_{N_1t}$ given $\mathbf{r}_{ft}$, which in turn can be factorized as the distribution of $\mathbf{r}_{N_2t}$ given $\mathbf{r}_{ft}$, times the distribution of $\mathbf{r}_{N_1t}$ given $\mathbf{r}_{N_2t}$ and $\mathbf{r}_{ft}$. Under the null, the conditional log-likelihood component will be (apart from constants):

$$-\frac{T}{2}\ln|\;{}_{N_2N_2}| - \frac{1}{2}\sum_{t=1}^{T} tr[\;{}^{-1}_{N_2N_2}(\mathbf{r}_{N_2t} - \mathbf{B}_{N_2}\mathbf{r}_{ft})(\mathbf{r}_{N_2t} - \mathbf{B}_{N_2}\mathbf{r}_{ft})']$$
$$-\frac{T_1}{2}\ln|\;{}^{*}_{N_{11}}| - \frac{1}{2}\sum_{t=1}^{T_1} tr[\;{}^{*-1}_{N_{11}}\mathbf{v}^{*}_{N_1t}\mathbf{v}^{*\prime}_{N_1t}]\} \qquad [17]$$

where ${}^{*}_{N_1N_1} = {}_{N_1N_1} - {}_{N_1N_2}\;{}^{-1}_{N_2N_2}\;{}_{N_2N_1}, \mathbf{v}^{*}_{N_1t} = \mathbf{r}_{N_1t} - \mathbf{B}^{*}_{N_1}\mathbf{r}_{ft} - \mathbf{B}^{*}_{N_1N_2}\mathbf{r}_{N_2t}$, $\mathbf{B}^{*}_{N_1N_2} = {}_{N_1N_2}\;{}^{-1}_{N_2N_2}$, and $\mathbf{B}^{*}_{N_1} = \mathbf{B}_{N_1} - \mathbf{B}^{*}_{N_1N_2}\mathbf{B}_{N_2}$.

With this one-to-one reparametrisation, it is clear that the ML estimates of $\ _{N_2N_2}$ and $\mathbf{B}_{N_2}$ can be obtained on the basis of the first line of [17] via N_2 OLS regressions of r_{it} $(i = N_1+1, \ldots, N)$ on $\mathbf{r}_{ft}$, whereas those of $\tilde{\ }^{*}_{N_1N_1}, \hat{\mathbf{B}}^{*}_{N_1}$ and $\hat{\mathbf{B}}^{*}_{N_1N_2}$ are obtained from another N_1 OLS regressions of r_{it} $(i = 1, \ldots, N_1)$ on $\mathbf{r}_{ft}$ and $\mathbf{r}_{N_2t}$. By virtue of the invariance property of ML estimation, $\hat{\mathbf{B}}_{N_1}$, $\tilde{\ }_{N_1N_1}$ and $\tilde{\ }_{N_1N_2}$ are then obtained by combining $\tilde{\ }_{N_2N_2}$ and $\hat{\mathbf{B}}_{N_2}$ with $\tilde{\ }^{*}_{N_1N_1}, \hat{\mathbf{B}}^{*}_{N_1}$ and $\hat{\mathbf{B}}^{*}_{N_1N_2}$

Since we can proceed along similar lines under the alternative, the contribution to the likelihood ratio test coming from the last N_2 assets will be $T(\ln|\tilde{\ }_{N_2N_2}| - \ln|\bar{\ }_{N_2N_2}|)$, and this is distributed as a χ^2 variable with N_2 degrees of freedom whether or not we estimate the model for the first N_1 assets. In fact, since the $\alpha's$ and $\beta's$ are asset specific, this is still true even if we consider a selectivity model for survivorship. Nevertheless, there is a loss of power, which is difficult to quantify, and depends on the size of the $\alpha_i's$ for the excluded assets.

TABLE 1
Individual Stocks and Value Weighted Index
Descriptive Statistics
Sample Period 1963:01-1992:12

Asset	Mean	Std. dev.
VW	0.4454	5.4198
Aguas Barcelona	0.6520	9.3007
Aguila	0.9177	11.8554
Altos Hornos Vizcaya	1.2770	14.1218
Asland	1.3623	11.4148
Aurora	0.6097	10.7410
General Azucarera	0.9130	9.9790
Banesto	1.2013	8.3255
Bilbao, Bilbao-Vizcaya	1.1824	7.6911
Guipuzcoano	0.8127	6.6159
Auxiliar FFCC	0.8288	13.1059
Carburos	0.9518	9.7146
Catalana Gas	0.7205	8.0897
Central	1.0484	7.3551
CEPSA	1.1870	9.0665

cont.

Table 1 (cont.)
Individual Stocks and Value Weighted Index
Descriptive Statistics
Sample Period 1963:01-1992:12

Asset	Mean	Std. dev.
CROS, ERCROS	1.1936	14.4584
Dragados	1.3910	11.3805
Ebro	0.8448	9.5048
Encinar	0.9507	10.2339
Energías Aragonesas	0.9348	11.3056
Española Zinc	0.8714	14.3543
Exterior	0.7489	7.4015
Fasa Renault	0.6592	11.7665
Felguera	1.2563	11.5467
Filipinas	0.8287	11.1857
Finanzauto	0.9246	10.0208
Fomento y Obras	1.1965	11.8851
Hidrola Cantábrico	0.9962	9.2615
Hidrola Cataluña	0.6218	7.2433
Iberduero	0.7488	9.8758
Lemona	0.8796	11.4024
MACOSA	0.8468	14.6708
NANSA	0.5972	8.6087
Papelera	1.5251	15.4168
Ponferrada	1.0278	14.6213
Popular	1.2202	8.1002
Reunidas	0.6935	9.3107
Sanson	0.8776	11.4718
Santana	0.7618	12.4386
Sefanitro	0.9774	18.1775
Sevillana	0.7122	6.7838
SNIACE	1.2665	14.2540
Tabacalera	0.8713	9.2049
Telefónica	0.9090	6.5636
Unión Eléctrica Fenosa	0.6934	6.9459
Urbis	1.2459	12.1566
Valderribas	1.0419	9.4148
Vallehermoso	1.0740	10.2344
VACESA	0.9399	8.5140

Table 1 presents the average excess return and standard deviation for the selected shares. The cross-sectional mean of the average excess returns is 0.787% (on a monthly basis), which is very similar to the equivalent statistic for the ten size-ranked portfolios. As expected, the main difference between both datasets is in terms of variability. Here, the smallest standard deviation is 6.563%, and the largest 18.177%, with a cross-sectional average of 10.564%, compared to a maximum standard deviation of 8.770% for the first decile portfolio.

FIGURE 1
VW portfolio excess returns

Figure 1 shows the returns on the VW index in excess of the safe asset. Apart from noticeable events, like the October 1987 crash, there are periods of high volatility followed by more quiet ones, which confirms the importance of modelling its time-variation, and suggests that there may be substantial differences between the traditional static approach and the conditional one that we propose. In fact, the changing nature of volatility would be even more noticeable if we considered weakly and daily returns. In this respect, it would certainly be desirable to use data at a higher frequency. Unfortunately, such data is only available for some aggregate indices, generally without adjustments for dividend payments or stock splits. Besides, since the Spanish stock market is relatively thin, with most trades concentrated in a few stocks, daily or weakly data could suffer from significant non-synchronous trading.

5. Empirical results

5.1. The unconditional evidence

As a benchmark, we present first the results obtained ignoring the dynamics in first and second moments. Assuming that the VW portfolio is a well-diversified basis portfolio, we have seen in Section 3.1.1 that estimators of a single factor model for excess returns with the asset pricing restrictions imposed can be obtained from the least squares regression of each asset return on the returns on the market portfolio. OLS estimators are efficient under the assumption that the conditional variances of the idiosyncratic terms are constant, and remains consistent when they follow univariate GQARCH processes instead. Furthermore, heteroskedasticity-robust standard errors are not required since the regressor is strongly exogenous under our assumptions. In this sense, the model is estimated exactly as a traditional CAPM. The parameter estimates are presented in Table 2.

TABLE 2
Individual Stocks
MARKET β's (Value weighted index)
OLS Regressions $r_{it} = \beta_i r_{VWt} + v_{it}$ $(i = 1, ..., 48)$
Sample Period 1963:01-1992:12

Asset	β	$(s.e.)$	R^2	s.e. reg.
Aguas Barcelona	0.8576	(0.0787)	0.2429	8.1131
Aguila	1.0753	(0.1000)	0.2437	10.3002
Altos Hornos Vizcaya	1.3180	(0.1180)	0.2617	12.1568
Asland	1.3464	(0.0852)	0.4070	8.7794
Aurora	0.4365	(0.1018)	0.0466	10.4892
General Azucarera	0.9310	(0.0839)	0.2511	8.6415
Banesto	1.1913	(0.0512)	0.5979	5.2774
Bilbao, Bilbao-Vizcaya	1.0928	(0.0484)	0.5833	4.9861
Guipuzcoano	0.8320	(0.0472)	0.4599	4.8636
Auxiliar FFCC	1.1440	(0.1124)	0.2199	11.5784
Carburos	0.9938	(0.0788)	0.3020	8.1213

(cont.)

TABLE 2(cont.)
Individual Stocks
MARKET β's (Value weighted index)
OLS Regressions $r_{it} = \beta_i r_{VWt} + v_{it}$ $(i = 1, ..., 48)$
Sample Period 1963:01-1992:12

Asset	β	$(s.e.)$	R^2	s.e. reg.
Catalana Gas	0.8676	(0.0647)	0.3280	6.6669
Central	0.9381	(0.0520)	0.4710	5.3611
CEPSA	1.2000	(0.0615)	0.5110	6.3372
CROS, ERCROS	1.5729	(0.1130)	0.3526	11.6451
Dragados	1.4121	(0.0823)	0.4461	8.4825
Ebro	0.8399	(0.0812)	0.2248	8.3708
Encinar	0.9344	(0.0874)	0.2359	9.0049
Energías Aragonesas	1.3168	(0.0849)	0.3992	8.7519
Española Zinc	1.0111	(0.1288)	0.1435	13.2755
Exterior	0.8173	(0.0580)	0.3516	5.9718
Fasa Renault	1.0101	(0.1012)	0.2134	10.4322
Felguera	1.2023	(0.0930)	0.3130	9.5811
Filipinas	0.7300	(0.1015)	0.1242	10.4553
Finanzauto	1.1034	(8.0766)	0.3512	8.0766
Fomento y Obras	1.1222	(0.0997)	0.2562	10.2680
Hidrola Cantábrico	0.9388	(0.0757)	0.2947	7.7965
Hidrola Cataluña	0.5391	(0.0823)	0.1637	6.6156
Iberduero	0.9316	(0.0823)	0.2608	8.4791
Lemona	0.9217	(0.1002)	0.1855	10.3219
MACOSA	1.2749	(12.9461)	0.2197	12.9461
NANSA	0.6397	(0.0769)	0.1567	7.9221
Papelera	1.5585	(0.1249)	0.3028	12.8651
Ponferrada	0.9255	(0.1331)	0.1178	13.7146
Popular	1.2086	(0.0474)	0.6422	4.8829
Reunidas	0.7256	(0.0820)	0.1756	8.4483
Sanson	0.7532	(0.1046)	0.1220	10.7745
Santana	1.2214	(0.1021)	0.2830	10.5178
Sefanitro	1.1484	(0.1657)	0.1159	17.0735
Sevillana	0.7264	(0.0537)	0.3337	5.5340
SNIACE	1.4933	(0.1135)	0.3278	11.6994
Tabacalera	0.9498	(0.0745)	0.3063	7.6780
Telefónica	0.8908	(0.0434)	0.5361	4.4690
Unión Eléctrica Fenosa	0.6749	(0.0573)	0.2752	5.9063

(cont.)

TABLE 2(cont.)
Individual Stocks
MARKET β's (Value weighted index)
OLS Regressions $r_{it} = \beta_i r_{VWt} + v_{it} \qquad (i = 1, ..., 48)$
Sample Period 1963:01-1992:12

Asset	β	$(s.e.)$	R^2	s.e. reg.
Urbis	1.4102	(0.0917)	0.3945	9.4472
Valderribas	1.0248	(0.0746)	0.3393	7.6833
Vallehermoso	1.3128	(0.0715)	0.4811	7.3632
VACESA	0.7916	(0.0721)	0.2460	7.4285

Tables 1 and 2 show the main advantage of working with individual shares instead of portfolios: there is substantial cross-sectional variation both in average returns and betas, with ranges (-0.3883, 1.54665), and (0.4364, 1.5718) respectively, as compared to (0.2946, 1.2644) and (0.9047, 1.1827) for decile portfolios. They also show, though, the main disadvantage: returns on individual stocks contain a substantial amount of idiosyncratic noise (about 2.3 times more on average). As a result, $R^{2\prime}s$ are more than halved, and standard errors for the coefficients more than doubled. Therefore, as far as power to reject the asset pricing restrictions is concerned, there is a trade-off between increasing the number and variety of the assets and having less precise estimates (see Rada and Sentana, 1997).

In terms of unconditional moments, the cross-sectional asset pricing restrictions amount to the ratio average risk premium/beta being the same for all assets, and equal to the average risk premium on the market as a whole. As we saw in Section 3.1.1, a simple, yet powerful way of testing such restrictions consists in including a constant in each of the 48 regressions, and checking if a significant coefficient is obtained (cf. Gibbons, Ross and Shanken, 1989). In this framework, the regression intercepts, known as Jensen's alphas in the portfolio evaluation literature, measure average "abnormal" returns from the point of view of the model. The estimated coefficients can be found in Table 3. The cross-sectional mean of the average abnormal returns is 0.3318, and some of them are much bigger in absolute value. However, the null hypothesis is rejected individually at conventional levels in

TABLE 3
Individual Stocks
Jensen's α's (Value weighted index)
OLS Regression $r_{it} = \alpha_i + \beta_i r_{VWt} + v_{it} \quad (i = 1, ..., 48)$
Sample Period 1963:01-1992:12

Asset	α	(s.e.)
Aguas Barcelona	0.7191	(0.4274)
Aguila	-0.3092	(0.5445)
Altos Hornos Vizcaya	-0.9820	(0.6408)
Asland	0.1784	(0.4642)
Aurora	0.5805	(0.5539)
General Azucarera	0.5560	(0.4560)
Banesto	0.2327	(0.2788)
Bilbao, Bilbao-Vizcaya	0.5310	(0.2622)
Guipuzcoano	0.2788	(0.2568)
Auxiliar FFCC	0.6652	(0.6113)
Carburos	0.5184	(0.4286)
Catalana Gas	0.7736	(0.3502)
Central	0.4512	(0.2825)
CEPSA	0.2745	(0.3348)
CROS, ERCROS	-0.8069	(0.6144)
Dragados	0.6429	(0.4473)
Ebro	0.4862	(0.4419)
Encinar	1.1382	(0.4724)
Energas Aragonesas	-0.1495	(0.4628)
Española Zinc	0.5099	(0.7015)
Exterior	0.4944	(0.3147)
Fasa Renault	0.4776	(0.5511)
Felguera	0.6809	(0.5054)
Filipinas	0.1821	(0.5528)
Finanzauto	0.5113	(0.4263)
Fomento y Obras	0.8133	(0.5413)
Hidrola Cantábrico	0.6755	(0.4108)
Hidrola Cataluña	-0.1119	(0.3498)
Iberduero	0.0049	(0.4484)
Lemona	0.9697	(0.5435)
MACOSA	0.3435	(0.6844)
NANSA	0.6609	(0.4175)

cont.

TABLE 3 (cont.)
Individual Stocks
Jensen's α's (Value weighted index)
OLS Regression $r_{it} = \alpha_i + \beta_i r_{VWt} + v_{it} \quad (i = 1, ..., 48)$
Sample Period 1963:01-1992:12

Asset	α	(s.e.)
Papelera	-0.5097	(0.6798)
Ponferrada	-0.0775	(0.7253)
Popular	0.6589	(0.2559)
Reunidas	0.3187	(0.4465)
Sanson	0.9302	(0.5677)
Santana	0.0349	(0.5562)
Sefanitro	0.4387	(0.9026)
Sevillana	0.2223	(0.2924)
SNIACE	-0.8280	(0.6172)
Tabacalera	0.5842	(0.4049)
Telefónica	0.2064	(0.2361)
Unión Eléctrica Fenosa	0.1072	(0.3123)
Urbis	0.0984	(0.4996)
Valderribas	0.7946	(0.4041)
Vallehermoso	0.1319	(0.3893)
VACESA	0.8265	(0.3904)

only five out of forty eight instances.

The joint tests, though, are more conclusive. Under the assumption of a diagonal idiosyncratic covariance matrix, the LR test yields 78.14, which is highly significant. The equivalent Wald test, which in this case is simply the sum of the squared t-ratios, is 78.47, while the LM test is 77.78 (cf. Berndt and Savin, 1977).[8] If we allow for non-zero covariances between idiosyncratic terms, parameter estimates and individual t-tests do not vary, but the joint Wald test is now 99.308 (p-value= 0.002%), so the null hypothesis that all in-

[8]We have repeated these tests using all 164 stocks. Under the assumption of a diagonal idiosyncratic covariance matrix, the joint likelihood ratio test is the sum of the LR tests of zero α_i for $i = 1, \ldots, 164$, whereas the Wald version is the sum of the individual squared t-ratios. They take the values 268.89 and 270.90 respectively, with 261.03 for the LM test, all of which are very highly significant.

tercepts are zero is clearly rejected. Hence, the results for individual shares confirm the existence of systematic biases in risk premia for the Spanish stock market, at least when a value weighted index is used as benchmark portfolio (see Rubio, 1988, and Sentana, 1995a).

In contrast, a test of zero price of risk for the common factor (i.e. $H_0 : \pi_1 = 0$) based on the sample mean of the excess returns on the VW portfolio only finds weak evidence against the null hypothesis (t-ratio=1.559, one-sided p-value=5.6%).

FIGURE 2
Empirical test of the CAPM using
VW portfolio as market portfolio

—— Security market line • Individual assets

The results of the individual tests are graphically represented in Figure 2, where we have plotted the average excess return for the 48 assets (cf. Table 1), and their market betas (cf. Table 2). In Figure 2, Jensen's alphas correspond to the vertical distances between the points that represent each portfolio, and the risk premia implied by the model, which lie on the security market line.

The joint test can also be represented graphically. Figure 3 shows

FIGURE 3
Empirical test of the CAPM using
VW portfolio as market portfolio

the position in mean-standard deviation space of the unconditional frontier for portfolios generated from the 48 excess returns with fixed weights. If the restrictions of the model were satisfied, the VW portfolio should coincide with the tangency portfolio.[9] However, it is clear that this is not the case. In this framework, the joint tests above examine if the ratio mean-standard deviation for the VW portfolio coincides with the same ratio for the tangency portfolio (see Gibbons, Ross and Shanken, 1989). In the portfolio evaluation jargon, the joint test checks if the Sharpe ratio for the VW portfolio is smaller than the maximum average return attainable per unit of risk.

Given that the safe asset used may not be very representative before 1982, we have also tested the zero beta version of the same model for gross returns in two different ways. First, we have estimated a model in which ν_0 is time-invariant. In this respect, ν_0 is estimated as 1.493% (s.e. 0.257%), under the assumption that the idiosyncratic covariance matrix is unrestricted. This is much larger than the average

[9]For this dataset, the tangency portfolio has a mean excess return of 2.250%, and a standard deviation of 6.103%.

return on the risk free rate (0.511%). The LR test for the hypothesis that the asset pricing restrictions are correct on average yields 77.02, which is significant at the 0.374% level. We have also considered a model with ν_{0t} allowed to change over time, which we estimated in terms of deviations of returns with respect to their cross-sectional average in each period. The Wald test for the hypothesis of unbiased pricing is 81.83 (p-value= 0.16%) when we allow for non-zero covariances between idiosyncratic terms, and 71.42 (p-value= 1.23%) when we do not. Therefore, we cannot reconcile theory and empirical evidence by replacing the safe asset with a zero-beta portfolio which plays a similar role.

It seems, therefore, that the VW index is not the best portfolio to proxy for the "market" in the CAPM sense; or in the context of our model, it may not be an appropriate representing portfolio for the common factor. This is confirmed when we estimate a model with two common factors, one observable and one unobservable. The latter does not affect the returns on the VW portfolio, which we still assume diversified, but it may affect the risk premia on the individual assets through the cross-sectional restriction in equation [7].[10] Using the estimation strategy set-up in Section 3.3, we find that the intercepts remain jointly significant (LR=77.10). Incidentally, it is worth mentioning that the estimate of the second factor, which is orthogonal to the VW portfolio by construction, has significant negative weightings on those firms with larger capitalizations (see the discussion below).

One potential solution would be to use alternative benchmark portfolios, such as an equally-weighted index. Another attractive possibility consists in avoiding the specification of the basis portfolio, and estimating a model with a single common factor by maximum likelihood. Here we shall follow this second route. Nevertheless, as we saw in Section 3.2 (see also Quah and Sargent, 1993), this approach is similar in spirit to repeating the analysis above using the Kalman

[10]Notice that if we set to zero the price of risk of the unobservable factor, we simply allow for a covariance structure for returns richer than the one considered in the CAPM model with diagonal idiosyncratic covariance matrix. Nevertheless, the betas and alphas in tables 3 and 4, as well as their standard errors, are numerically identical. The joint test is obviously affected, but only marginally so (LR=78.42).

TABLE 4
Average Weights
Based on OLS regressions of r_{VWt} and
the factor estimate on r_{it} (i=1,...,48)
Sample Period 1963:01-1992:12

Asset	VW	Unobservable factor
Aguas Barcelona	-0.1556	1.8209
Aguila	0.7942	1.4694
Altos Hornos Vizcaya	0.5411	1.3888
Asland	2.1440	2.5679
Aurora	-0.7055	0.5359
General Azucarera	0.0522	1.7113
Banesto	6.8187	3.8079
Bilbao, Bilbao-Vizcaya	9.6228	3.9472
Guipuzcoano	3.3640	4.2054
Auxiliar FFCC	1.3374	1.2488
Carburos	2.1957	2.1773
Catalana Gas	-1.8462	3.0933
Central	8.7833	3.1256
CEPSA	2.0201	4.5331
CROS, ERCROS	1.7207	1.6948
Dragados	1.0366	3.1511
Ebro	1.7264	1.4753
Encinar	1.2417	1.7318
Energas Aragonesas	0.7824	2.7186
Española Zinc	-0.2733	0.7170
Exterior	2.8641	2.7084
Fasa Renault	2.8724	1.3673
Felguera	1.1359	1.7109
Filipinas	-0.0918	0.9531
Finanzauto	1.2035	2.6719
Fomento y Obras	0.2695	1.4819
Hidrola Cantábrico	1.3096	1.9370
Hidrola Cataluña	-0.6037	1.3327
Iberduero	7.2553	1.0307
Lemona	1.2351	1.1621
MACOSA	1.0975	1.1677

(cont.)

Table 4(cont.)
Average Weights
Based on OLS regressions of r_{VWt} and
the factor estimate on r_{it} (i=1,...,48)
Sample Period 1963:01-1992:12

Asset	VW	Unobservable factor
NANSA	0.0886	1.1684
Papelera	0.1143	1.6815
Ponferrada	-0.7566	0.7535
Popular	4.7153	5.0553
Reunidas	0.1315	1.2066
Sanson	-1.6243	1.0324
Santana	-0.4088	1.6152
Sefanitro	-0.3959	0.5907
Sevillana	5.8934	2.2154
SNIACE	1.7454	1.5773
Tabacalera	2.6686	2.0208
Telefónica	16.4559	3.7399
Unión Eléctrica Fenosa	8.0397	1.6173
Urbis	-0.2053	2.6383
Valderribas	2.5783	2.4180
Vallehermoso	0.0603	4.0567
VACESA	1.1513	1.9674

filter estimate of the common factor as the benchmark portfolio.

The assumption of diagonal idiosyncratic covariance matrix crucially determines the implicit factor representing portfolio,[11] but imposes many overidentifying restrictions on the unconditional covariance matrix of excess returns. Strictly speaking, identification could be achieved for a finite number of assets by a nondiagonal $_N$ matrix of rank $N-1$. Under such an assumption, though, we could always fit the vector of means and covariance matrix for excess returns perfectly with a single factor model, provided that we choose as factor representing portfolio the tangency portfolio in Figure 3.

[11]Such a portfolio can also be understood as the one that best explains the covariances between the asset returns (see Sentana and Shah, 1994).

Although its composition is in principle different, the excess return on the estimated basis portfolio is rather similar to the VW portfolio, with a correlation coefficient of 0.939.[12] Furthermore, it is also very highly correlated (0.980) with the factor representing portfolio estimated from the ten size-ranked portfolios in Sentana (1995a). A convenient way of interpreting the common factor can be obtained by regressing the estimated factor on the set of stock returns. Since we are assuming that the variances and covariances of the asset returns are constant over time, so are the weights used by the Kalman filter to estimate the factor, and hence, the regression R^2 is 1. However, if we compute an analogous regression for the VW index, the R^2 will not be exactly 1, as the VW index do not have fix weights, although the approximation is rather good ($R^2 = 0.975$). The average weights obtained in this way can be found in Table 4. As expected, the VW portfolio mainly represents firms with larger capitalizations, such as Banesto, BBV, Central, Iberduero, Popular, Sevillana, Fenosa and especially Telefónica. By contrast, the weightings for the estimated factor are more evenly distributed, although they are far from corresponding to those of an equally-weighted index.

The results are presented in Table 5 for the normalization $\beta_{48} = 1$. Note that the price of common risk is positive and significantly different from zero. If we include a constant in the equations for the first forty seven assets, and normalize with $\alpha_{48} = 0$, the results in Table 6 show that the intercepts are not jointly significant (LR= 51.18, p-value= 31.30%), and only individually significant in four cases.[13] If we normalize the factor representing portfolio so that its weights add up to 1, its expected excess return is estimated to be 0.8482%, with a standard deviation of 6.1995%. Hence, it lies somewhat closer to the tangency portfolio than the VW index; close enough, it seems, to avoid the test rejection.

We have also tested zero-beta versions of this model for gross returns.

[12]The estimation of the factor is rather accurate in the sense that its estimated mean square error is only 3.6% of its variance.

[13]Unlike the joint test, the individual tests are not invariant to the normalization used. With $\alpha_l = 0$ and $\beta_l = 1$, the estimate of π is the sample mean of r_{lt}, whereas the estimate of α_i^* is the sample mean of r_{it} minus its estimated beta times the sample mean of r_{lt}.

Table 5
Individual Stocks
Unobservable factor β's MLE $r_{it} = \tau\lambda\beta_i + \beta_i f_t + v_{it}$ $(i = 1, ..., 48; \beta_{48} = 1)$
Sample Period 1963:01-1992:12

Asset	β	$(s.e.)$	R^2	Idiosyncratic std. dev.
Aguas Barcelona	1.0945	(0.1069)	0.2856	7.8614
Aguila	1.4004	(0.1352)	0.3015	9.9085
Altos Hornos Vizcaya	1.7732	(0.1626)	0.3398	11.4744
Asland	1.7041	(0.1297)	0.4756	8.2662
Aurora	0.5669	(0.1240)	0.0573	10.4287
General Azucarera	1.1753	(0.1157)	0.2905	8.4052
Banesto	1.2802	(0.0961)	0.5010	5.8813
Bilbao, Bilbao-Vizcaya	1.1729	(0.0887)	0.4837	5.5266
Guipuzcoano	0.9728	(0.0758)	0.4563	4.8781
Auxiliar FFCC	1.5060	(0.1509)	0.2777	11.1387
Carburos	1.2745	(0.1123)	0.3618	7.7606
Catalana Gas	1.1505	(0.0931)	0.4157	6.1835
Central	1.0050	(0.0849)	0.3889	5.7496
CEPSA	1.5041	(0.1033)	0.5845	5.8444
CROS, ERCROS	2.0074	(0.1636)	0.4157	11.0521
Dragados	1.8235	(0.1318)	0.5404	7.7157
Ebro	0.9971	(0.1084)	0.2304	8.3381
Encinar	1.2427	(0.1184)	0.2958	8.5877
Energas Aragonesas	1.7117	(0.1297)	0.4923	8.0553
Española Zinc	1.2171	(0.1645)	0.1524	13.2157
Exterior	0.9495	(0.0858)	0.3420	6.0039
Fasa Renault	1.3371	(0.1342)	0.2732	10.0311
Felguera	1.4680	(0.1333)	0.3380	9.3945
Filipinas	0.9708	(0.1282)	0.1621	10.2390
Finanzauto	1.4525	(0.1155)	0.4430	7.4790
Fomento y Obras	1.4264	(0.1360)	0.2993	9.9490
Hidrola Cantábrico	1.1239	(0.1061)	0.3044	7.7242
Hidrola Cataluña	0.5831	(0.0833)	0.1413	6.7121
Iberduero	0.8285	(0.1133)	0.1518	9.0956
Lemona	1.1607	(0.1318)	0.2103	10.1330
MACOSA	1.7229	(0.1675)	0.2944	12.3236
NANSA	0.7228	(0.0986)	0.1419	7.9747

(cont.)

TABLE 5 (cont.)
Individual Stocks
Unobservable factor β's MLE $r_{it} = \tau\lambda\beta_i + \beta_i f_t + v_{it}$ $(i = 1, ..., 48; \beta_{48} = 1)$
Sample Period 1963:01-1992:12

Asset	β	$(s.e.)$	R^2	Idiosyncratic std. dev.
Papelera	2.1902	(0.1769)	0.4350	11.5881
Ponferrada	1.2970	(0.1664)	0.1708	13.3139
Popular	1.3548	(0.0934)	0.5804	5.2473
Reunidas	0.8410	(0.1066)	0.1728	8.4680
Sanson	1.0819	(0.1311)	0.1813	10.3800
Santana	1.5831	(0.1414)	0.3476	10.0466
Sefanitro	1.5868	(0.2091)	0.1633	16.6271
Sevillana	0.7412	(0.0776)	0.2521	5.8668
SNIACE	1.9074	(0.1626)	0.3863	11.1668
Tabacalera	1.1364	(0.1057)	0.3174	7.6048
Telefónica	0.9182	(0.0761)	0.4136	5.0261
Unión Eléctrica Fenosa	0.6266	(0.0796)	0.1737	6.3140
Urbis	1.8698	(0.1398)	0.5060	8.5441
Valderribas	1.2843	(0.1087)	0.3839	7.3896
Vallehermoso	1.6988	(0.1164)	0.5882	6.5678
VACESA	1.0000	(0.1164)	0.2793	7.2279
τ	0.0299	(0.0119)		

When we impose the extra restriction that the expected return on the zero-beta portfolio is constant, ν_0 is estimated as 1.313%. The likelihood ratio test for unbiased pricing yields 36.64, which is only significant at the 83.65% level. A similar result is obtained when we allow ν_{0t} to change over time, and estimate the model in terms of deviations of returns with respect to their cross-sectional average (LR=39.54, p-value = 86.19%).

Therefore, if we concentrate on the unconditional moments of returns, our results for individual shares confirm the existence of systematic biases in average risk premia for the Spanish stock market when a value weighted index is used as the benchmark portfolio. However, we are unable to reject the same null hypothesis in a model with a single unobservable factor. This is true for versions of the models

Table 6
Individual Stocks
Jensen's α's (Unobservable factor)
MLE $r_{it} = \alpha_i + \tau\lambda\beta_i + \beta_i f_t + v_{it}$ $(i = 1, ..., 48; \beta_{48} = 1; \alpha_{48} = 0)$
Sample Period 1963:01-1992:12

Asset	α	(s.e.)
Aguas Barcelona	-0.1955	(0.6609)
Aguila	-1.5270	(0.8288)
Altos Hornos Vizcaya	-2.5644	(1.0331)
Asland	-1.2684	(0.9244)
Aurora	0.1104	(0.6054)
General Azucarera	-0.4279	(0.7039)
Banesto	-0.7664	(0.6575)
Bilbao, Bilbao-Vizcaya	-0.3746	(0.6015)
Guipuzcoano	-0.5115	(0.5045)
Auxiliar FFCC	-0.6203	(0.9207)
Carburos	-0.5600	(0.7371)
Catalana Gas	-0.2038	(0.6316)
Central	-0.3231	(0.5352)
CEPSA	-0.9935	(0.7659)
CROS, ERCROS	-2.5532	(1.1180)
Dragados	-0.9045	(0.9369)
Ebro	-0.3260	(0.6394)
Encinar	0.0906	(0.7546)
Energas Aragonesas	-1.6318	(0.9280)
Española Zinc	-0.4902	(0.9025)
Exterior	-0.2678	(0.5189)
Fasa Renault	-0.6694	(0.8370)
Felguera	-0.5298	(0.8502)
Filipinas	-0.6573	(0.7154)
Finanzauto	-0.7320	(0.8075)
Fomento y Obras	-0.3788	(0.8255)
Hidrola Cantábrico	-0.2389	(0.6710)
Hidrola Cataluña	-0.5763	(0.4460)
Iberduero	-0.5732	(0.6215)
Lemona	0.0114	(0.7707)
MACOSA	-1.1562	(1.0591)
NANSA	0.0959	(0.5451)

cont.

TABLE 6(cont.)
Individual Stocks
Jensen's α's (Unobservable factor)
MLE $r_{it} = \alpha_i + \tau\lambda\beta_i + \beta_i f_t + v_{it}$ $(i = 1, ..., 48; \beta_{48} = 1; \alpha_{48} = 0)$
Sample Period 1963:01-1992:12

Asset	α	(s.e.)
Papelera	-2.4761	(1.2149)
Ponferrada	-1.2343	(0.9455)
Popular	-0.4102	(0.6703)
Reunidas	-0.3606	(0.5960)
Sanson	-0.0106	(0.7386)
Santana	-1.3282	(0.9307)
Sefanitro	-0.9525	(1.1915)
Sevillana	-0.3372	(0.4604)
SNIACE	-2.4888	(1.1116)
Tabacalera	-0.3423	(0.6638)
Telefónica	-0.4934	(0.4976)
Unión Eléctrica Fenosa	-0.3407	(0.4468)
Urbis	-1.5242	(0.9996)
Valderribas	-0.2715	(0.7063)
Vallehermoso	-1.3257	(0.8734)
VACESA	0	

with and without a safe asset.

5.2. The conditional evidence

In this section, we examine the effect of the ignored time-variation in risk premia and the conditional covariance matrix of returns on the estimation and testing of the different versions of the model. At the same time, the emphasis on conditional moments allows us to test whether the model restrictions are satisfied not only on average, but also over time. For instance, apart from checking if Jensen's alphas are 0, we can also test hypothesis related to the integration or segmentation of the Spanish stock market. In particular, we can test if the price of common risk is the same for all assets, and also, if asset risk premia depend on the volatility of their idiosyncratic terms, ω_{iit}. Therefore, it is possible in this framework to distinguish empirically between the differential valuation of common risk and the valuation of idiosyncratic risk, which is impossible in an unconditional setting.

TABLE 7
Value Weighted Excess Returns
GQARCH(1,1)-M Parameter Estimates and Standard Errors
Sample Period 1963:01-1992:12

r_{VWt}	$= 0.00651\lambda_t + f_t$			
	(0.09687)			
λ_t	$= 0.05039$	$+0.11256\ f_{t-1}$	$+0.06286\ f_{t-1}^2$	$+0.93034\ \lambda_{t-1}$
		(0.03906)	(0.02222)	(0.02342)

FIGURE 4
Conditional standard deviation
VW portfolio

As we discussed in Section 3.1.1, given our maintained assumptions, efficient estimates of the parameters of a conditional factor model with a single observable factor for excess returns can be obtained from a univariate GQARCH-M model for the VW portfolio, together with univariate GQARCH regressions of the returns on the different assets on the returns on the VW index. The parameter estimates obtained in this way can be found in tables 7 and 8, while the estimated conditional standard deviation of the market portfolio is represented in Figure 4. At a purely descriptive level, it is worth mentioning the very high degree of persistence in volatility, as measured by the sum of the ARCH and GARCH coefficients, and also the asymmetric response of volatility to positive and negative shocks of the same size. However, the asymmetric effect is the opposite to the one found for US and UK returns, in that positive shocks seem to have a larger impact than negative ones in the Spanish stock market.

Turning now to the market price of risk, notice that it is positive but not significantly different from zero. This result is similar to the one obtained by Alonso and Restoy (1995) for the Spanish portfolio in the Morgan-Stanley database with different conditional variance specifications.

The estimated betas in Table 8 are different from the ones obtained by ordinary least squares in Table 2 (correlation coefficient=0.804), and significantly so in some cases. Such changes are mainly due to the substantial degree of time-variation in the volatility of idiosyncratic components, which imply a substantive difference between the ordinary least squares used in Section 5.1 and the "weighted" least squares implicit in the maximum likelihood estimation of the GQARCH regressions.

Unlike in Sentana (1995a), the changes in estimated betas seem to drive away the significance of most of the alphas in Table 9. However, note that the alpha for the market portfolio is significant. The joint tests produce mixed results: while the LR test is 53.22, with a p-value of 28.02%, the LM test takes the value 66.36 (p-value= 4.05%). Therefore, the evidence against the null hypothesis of unbiased pricing is weaker in the conditional setting.

This is in contrast with the results obtained for size-ranked portfolios. Two reasons could account for the difference. First, we conjectured in Sentana (1995a) that the rejection of the asset pricing restrictions could be due to the behaviour of share prices of smaller firms, with low volume and frequency of trading. Since the firms that stay in the sample since 1963 until 1992 include many larger, well-known

TABLE 8
Individual Stocks
MARKET β's (Value weighted index)
FIML $r_{it} = \beta_i r_{VWt} + v_{it}$ $(i = 1, ..., 48)$; $v_{it}|I_{t-1} \sim N(0, \omega_{iit})$
Sample Period 1963:01-1992:12

Asset	β	(s.e.)
Aguas Barcelona	0.6519	(0.0589)
Aguila	0.9177	(0.0856)
Altos Hornos Vizcaya	1.2770	(0.0988)
Asland	1.3623	(0.0859)
Aurora	0.6097	(0.1071)
General Azucarera	0.9130	(0.0834)
Banesto	1.2013	(0.0586)
Bilbao, Bilbao-Vizcaya	1.1824	(0.0515)
Guipuzcoano	0.8127	(0.0460)
Auxiliar FFCC	0.8288	(0.1157)
Carburos	0.9518	(0.0804)
Catalana Gas	0.7205	(0.0643)
Central	1.0484	(0.0625)
CEPSA	1.1870	(0.0550)
CROS, ERCROS	1.1936	(0.1327)
Dragados	1.3910	(0.0873)
Ebro	0.8448	(0.0840)
Encinar	0.9507	(0.0761)
Energas Aragonesas	0.9348	(0.0699)
Española Zinc	0.8714	(0.1133)
Exterior	0.7489	(0.0669)
Fasa Renault	0.6592	(0.1099)
Felguera	1.2563	(0.0952)
Filipinas	0.8287	(0.1010)
Finanzauto	0.9246	(0.1072)
Fomento y Obras	1.1965	(0.0934)
Hidrola Cantábrico	0.9962	(0.0502)
Hidrola Cataluña	0.6218	(0.0467)
Iberduero	0.7488	(0.0557)
Lemona	0.8796	(0.0859)
MACOSA	0.8468	(0.0820)
NANSA	0.5972	(0.0765)

(cont.)

TABLE 8(cont.)
Individual Stocks
MARKET β's (Value weighted index)
FIML $r_{it} = \beta_i r_{VWt} + v_{it}$ $(i = 1, ..., 48)$; $v_{it}|I_{t-1} \sim N(0, \omega_{iit})$
Sample Period 1963:01-1992:12

Asset	β	(s.e.)
Papelera	1.5251	(0.1114)
Ponferrada	1.0278	(0.1078)
Popular	1.2202	(0.0542)
Reunidas	0.6935	(0.0749)
Sanson	0.8776	(0.0911)
Santana	0.7618	(0.0961)
Sefanitro	0.9774	(0.1273)
Sevillana	0.7122	(0.0516)
SNIACE	1.2665	(0.1013)
Tabacalera	0.8713	(0.0786)
Telefónica	0.9090	(0.0432)
Unión Eléctrica Fenosa	0.6934	(0.0600)
Urbis	1.2459	(0.0806)
Valderribas	1.0419	(0.0759)
Vallehermoso	1.0740	(0.0677)
VACESA	0.9399	(0.0680)

firms, with high volume and frequency of trading, our results could be interpreted as providing some evidence for such a conjecture. The second reason, though, is that by working with individual returns, the trade-off between increasing the number and variety of the assets and having less precise estimates results in a substantial loss of power in our case (see Rada and Sentana, 1997).

But as we said before, in a conditional setting we can also test if the model is rejected in other directions more informative from an economic point of view. First, we can test whether unsystematic risk, as measured by the asset-specific conditional variance, ω_{iit}, affects risk premia. The LM version of such a test is particularly simple. For each asset, we regress the standardized residuals $\hat{\omega}_{iit}^{-1/2}(r_{it} - \hat{\beta}_i r_{VWt})$ on $\hat{\omega}_{iit}^{-1/2} r_{VWt}$ and $\hat{\omega}_{iit}^{1/2}$, where ˆ means ML estimate under the null, and compute the LM test as TR^2 (or $(T-2)R^2/(1-R^2)$ in its F-

Table 9
Individual Stocks and Value Weighted Index
Jensen's α's (Value weighted index)
FIML $r_{it} = \alpha_i + \beta_i r_{VWt} + v_{it} (i = 1, ..., 48);\ v_{it} \mid I_{t-1} \sim N(0, \omega_{iit})$
$r_{VWt} = \alpha_{vw} + \tau\lambda_t + f_t \qquad f_t \mid I_{t-1} \sim N(0, \lambda_t)$
Sample Period 1963:01-1992:12

Asset	α	(s.e.)
Aguas Barcelona	0.0008	(0.2556)
Aguila	0.5298	(0.3394)
Altos Hornos Vizcaya	0.6820	(0.4240)
Asland	0.0410	(0.3968)
Aurora	0.8080	(0.4720)
General Azucarera	0.0697	(0.3285)
Banesto	0.2197	(0.2473)
Bilbao, Bilbao-Vizcaya	0.4463	(0.2252)
Guipuzcoano	0.1230	(0.2069)
Auxiliar FFCC	0.3774	(0.4393)
Carburos	0.2504	(0.3458)
Catalana Gas	0.5972	(0.2906)
Central	0.5284	(0.2091)
CEPSA	0.1476	(0.3050)
CROS, ERCROS	0.6240	(0.4833)
Dragados	0.6437	(0.4057)
Ebro	0.2910	(0.3166)
Encinar	0.7103	(0.4048)
Energas Aragonesas	0.3063	(0.2928)
Española Zinc	0.1440	(0.4040)
Exterior	0.4584	(0.2869)
Fasa Renault	0.0150	(0.3829)
Felguera	0.3982	(0.4541)
Filipinas	0.0902	(0.3963)
Finanzauto	0.5238	(0.3513)
Fomento y Obras	0.4456	(0.4372)
Hidrola Cantábrico	0.5034	(0.2961)
Hidrola Cataluña	0.1028	(0.2001)
Iberduero	0.0650	(0.2875)
Lemona	0.2705	(0.4345)
MACOSA	0.2794	(0.3305)

cont.

Table 9(cont.)
Individual Stocks and Value Weighted Index
Jensen's α's (Value weighted index)
FIML $r_{it} = \alpha_i + \beta_i r_{VWt} + v_{it} (i = 1, ..., 48);\ v_{it} \mid I_{t-1} \sim N(0, \omega_{iit})$
$r_{VWt} = \alpha_{vw} + \tau\lambda_t + f_t \qquad f_t \mid I_{t-1} \sim N(0, \lambda_t)$
Sample Period 1963:01-1992:12

Asset	α	(s.e.)
NANSA	0.3517	(0.3113)
Papelera	0.6379	(0.5136)
Ponferrada	0.3513	(0.5326)
Popular	0.6757	(0.2470)
Reunidas	0.1120	(0.3071)
Sanson	0.2658	(0.3886)
Santana	0.0012	(0.3744)
Sefanitro	0.2520	(0.5416)
Sevillana	0.2245	(0.1926)
SNIACE	1.0030	(0.4373)
Tabacalera	0.3622	(0.3590)
Telefónica	0.2338	(0.2220)
Unión Eléctrica Fenosa	0.1533	(0.2214)
Urbis	0.1319	(0.3452)
Valderribas	0.4345	(0.3172)
Vallehermoso	0.1431	(0.2976)
VACESA	0.7524	(0.3268)
VW	0.6086	(0.2015)

version). We find that, without exception, we cannot reject the null hypothesis that idiosyncratic risk is not priced. Under the assumption of diagonal idiosyncratic covariance matrix, the joint LM test, which is simply the sum of the individual ones, yields 61.55, with a p-value of 9.06%. This result is confirmed when we estimate GQARCH-M regressions of each asset excess returns on the excess returns of the VW portfolio, and compute the sum of the individual likelihood ratio tests (joint LR=43.48, p-value=65.84%).

We can also test whether systematic risk is valued differently across assets by allowing τ to vary with i. The evidence here is less supportive of the model restrictions. The LM version of such a test yields

61.01, with a p-value of 9.85%. However, when we add the estimated conditional variance of the VW portfolio, $\hat{\lambda}_{VWt}$, as an extra explanatory variable in each of the 48 individual GQARCH regressions, the joint LR test is 105.86 (p-value$\simeq$ 0).

If we test the zero-beta version of the same model in terms of deviations of returns with respect to their cross-sectional average in each period, the results worsen. The LM and LR versions of the test for the hypothesis of unbiased pricing are 79.87 and 71.30 (p-values 0.27% and 1.61% respectively). Similarly, the hypothesis that unsystematic risk should not affect risk premia is also rejected at conventional levels (LM=72.14, LR=64.68, p-values 1.36% and 5.44% respectively). Finally, the evidence against common risk pricing is even stronger (LM=107.68, LR=132.3, p-values$\simeq$ 0).

TABLE 10
Individual Stocks
(Unobservable Factor)
FIML $r_{it} = \tau\lambda_t\beta_i + \beta_i f_t + v_{it}$ $(i = 1, ..., 10; \beta_{48} = 1)$
$f_t \mid I_{t-1} \sim N(0, \lambda_t)$; $v_{it} \mid I_{t-1} \sim N(0, \omega_{iit})$

	Asset Pricing Tests
1	Is systematic risk significantly priced? LR=2.10 (p-value=14.73%)
2	Is systematic risk priced differently across assets? LR=135.74(p-value=0)
3	Is idiosyncratic risk priced? LR=58.90 (p-value=13.46%)
4	Does the model price assets correctly on average? LR=63.34 (p-value=6.80%)

We have also considered a conditional version of the model with an unobservable factor, in which both common and idiosyncratic factors are allowed to have time-varying conditional variances of the univariate GQARCH(1,1) type. The results are in broad agreement with the ones obtained for the model with an observable factor. The test statistics presented in Table 10 show that the price of systematic risk,

TABLE 11
Individual Stocks
(Unobservable Factor)

FIML $R_{it} - \overline{R}_t = \tau\lambda_t(\beta_i - \overline{\beta}) + (\beta_i - \overline{\beta})f_t + v_{it} \quad (i = 1, ..., 10; \beta_{48} - \overline{\beta} = 1)$

$f_t \mid I_{t-1} \sim N(0, \lambda_t); \quad v_{it} \mid I_{t-1} \sim N(0, \omega_{iit})$

	Asset Pricing Tests
1	Is systematic risk significantly priced? LR=0.40 (p-value=52.71%)
2	Is systematic risk priced differently across assets? LR=234.76(p-value=0)
3	Is idiosyncratic risk priced? LR=38.88 (p-value=82.33%)
4	Does the model price assets correctly on average? LR=62.56 (p-value=7.71%)

τ, although positive, does not seem to be significant. More importantly, they also indicate that the price of risk does not seem to be common. However, the evidence against unbiased pricing, like in the unconditional setting, is weaker (p-value= 6.80%). The same is true for the restriction that idiosyncratic risks, ω_{iit}, do not affect risk premia (p-value= 13.46%). These results are confirmed in the zero beta version of the same model, which we estimate in terms of deviations of returns with respect to their cross-sectional average in each period (see Table 11).

6. Conclusions

In this paper we use monthly return data on individual firms and a capitalization-weighted index for the Spanish stock market for the period January 1963 - December 1992. We test the restrictions implied by different versions of a dynamic APT-type asset pricing model on the risk-return relationship.

When we look at the unconditional moments of returns, our results for individual shares confirm the existence of systematic biases in average risk premia for the Spanish stock market, at least when a value weighted index is used as the benchmark portfolio. This is true for versions of the model with and without a safe asset. However, we are unable to reject the same null hypothesis in an APT-type model with a single unobservable factor.

When we take into account the time-variation in the covariance matrix of asset returns, we find that if we use the value weighted index as the benchmark portfolio, the evidence against the null hypothesis of unbiased pricing is weaker for the 48 firms with data for the whole sample period. We also find that the idiosyncratic risk components do not seem to be rewarded. However, we find some evidence against the hypothesis that the price of systematic risk is common for all shares. The evidence against the model restrictions is significantly stronger when we test the zero-beta version of the same model in terms of deviations of returns with respect to their cross-sectional average in each period.

Similar results are obtained with a conditional version of a single factor dynamic-APT. Although the evidence against unbiased pricing and for the pricing of diversifiable idiosyncratic risk is weak, there is strong evidence for the view that the price of systematic risk is not common. This is true for versions of the model with and without a safe asset.

Therefore, it seems that the restriction of common risk pricing implied by our theoretical model is not supported by the data, irrespectively of whether the common factor is observable or unobservable, and irrespectively of whether or not there is a riskless asset available.

Obviously, as is true of virtually all econometric tests of theoretical restrictions, we are testing not only those restrictions, but also all the maintained assumptions that underlie our intertemporal asset pricing model and its empirical implementation. In particular, the assumptions of constant factor loadings, and especially, constant price of risk are, no doubt, potentially restrictive. Searching for a more flexible parametrisation provides an interesting avenue for further research.

Finally, it is worth mentioning that we have ignored potential seasonal effects, such as the so-called "January effect", which Basarrate and Rubio (1994) have shown is also relevant in the Spanish stock market.

An interesting possibility within our framework would be to estimate a dynamic APT model with seasonal factors to see if we can explain such an effect in terms of seasonality in the risk return relationship (see Demos, Sentana and Shah, 1993).

Appendix

Optimal selection of a balanced panel from an unbalanced one

A dataset is called a panel when we have time series observations on a sample of individual units. A panel is said to be (contemporaneously) balanced if for each of the N individual units we have T time series observations over the same historic period, or analogously, if for each of the T sample periods, we have cross-sectional observations for all N individual units. Nevertheless, most data panels are incomplete or unbalanced, in the sense that the time span differs for different individual units, or analogously, the individual units on which we have observations change over time.

Although conceptually there is little difference between a balanced and an unbalanced panel, especially if units get in and out of the sample at random, from a computational point of view, many econometric procedures are considerably simplified when we work with a balanced panel. Therefore, it is not surprising that many empirical studies end up balancing the dataset, and reduce the number of available observations, either in the time series dimension, or in the cross-sectional dimension or both. The purpose of this appendix is to develop a selection procedure to balance an incomplete panel in a well defined optimal sense.

In principle, it would be desirable to have a dataset with a time span as large as possible, so that the number of effective time periods, T_e is high. For this reason, it is common to retain only those individual units with observations in all possible time periods. At the same time, it is also desirable that the number of individual units retained, N_e, is as large as possible. One way to achieve this is to retain those sample periods for which we have observations on all units, so that $N_e = N$. Unfortunately, there is usually a trade off between both objectives. For a given initial period, the number of individual units for which there are consecutive observations decreases as we include more and more periods.

As a consequence, to balance a panel in an optimal sense, it is necessary to maximize implicitly or explicitly a "utility" function, $U()$, defined over pairs N_e, T_e. For instance, in the examples above we had lexicographic preferences over T_e and then N_e, or over N_e and then T_e. A less extreme case is to have $U(N_e, T_e) = N_e T_e$ - Number of parameters = degrees of freedom. To carry out the optimization it is necessary to take into account potential restrictions over the domain of T_e, N_e. For instance, we may want to include one or several specific periods, or one or several specific individual units.

Nevertheless, it is convenient to consider first all possible unconstrained combinations of N_e, T_e and initial period. A simple way to do it is to compute a $T \times T$ matrix, $\mathbf{N}$, such that $\{\mathbf{N}\}_{ts}$ indicates the number of individual units in the sample continuously from at least period t until at least period s. Such a matrix satisfies two convenient properties which simplify the sample selection. First, we can concentrate on its upper triangle. Second, given that the number of periods between t and s is constant along each diagonal ($T_e = \mid s - t \mid +1$), if our objective function is nondecreasing in N_e, we can restrict our attention to the largest $\{\mathbf{N}\}_{ts}$ along each diagonal. It is perhaps convenient to represent the matrix $\mathbf{N}$ graphically:

$$
\begin{array}{rllllll}
s & 1 & 2 & 3 & \dots & T-1 & T \\
t & & & & & & \\
1 & N_{11} & N_{12} & N_{13} & \dots & N_{1T-1} & N_{1T} \\
2 & & N_{23} & N_{23} & \dots & N_{2T-1} & N_{2T} \\
3 & & & N_{33} & \dots & N_{3T-1} & N_{3T} \\
\vdots & & & & \ddots & \ddots & \\
T-1 & & & & & N_{T-1T-1} & N_{T-1T} \\
T & & & & & & N_{TT}
\end{array}
$$

Therefore, the pairs that form the effective choice set will be T_e and the maximum of N_{ts} along the corresponding diagonal.

The restricted case is fairly easy to analyze in this framework. If a particular period has to be necessarily included, say t_1, then we should restrict our attention to those elements of the matrix $\mathbf{N}$ above and to the right of element t_1, t_1, including $N_{t_1 t_1}$ itself. Analogously, if we want to exclude a particular period, say t_2, then we must only look at those elements strictly below and to the left of $N_{t_2 t_2}$. On the other hand, if we want to include always a given unit, say i, on which we have observations from period t_i to period s_i, then we must exclude all periods before t_i and after s_i. By contrast, if we want to exclude some specific individual units, the best way to proceed is to recompute $\mathbf{N}$ by subtracting for each unit, say j, one from the square $(s_j - t_j + 1)$ principal minor starting in $N_{t_j t_j}$ and finishing in $N_{s_j s_j}$. Obviously we can have simultaneous restrictions, in which case our choice set is the intersection of the choice sets for each individual restriction. Again, we could first maximize along each diagonal within the globally restricted choice set.

A practical problem may arise if the time series for an individual unit has missing values. To balance the panel, it is usually more convenient to consider a different "new" unit for each interval with consecutive observations.

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Resumen

En este trabajo se usan rentabilidades mensuales de acciones de empresas individuales cotizadas en el mercado de valores español entre 1963 y 1992 para contrastar si se cumplen las restricciones que modelos APT dinámicos imponen a la relación riesgo-rentabilidad.

Los resultados sugieren la existencia de sesgos sistemáticos en las primas de riesgo medias cuando se usa como factor observable un índice de mercado ponderado por capitalización, pero no cuando el factor se deja sin especificar. Si se tiene en cuenta la variación temporal en la matriz de varianzas y covarianzas de las rentabilidades, se obtiene que el "precio del riesgo" no es común para los distintos activos, independientemente de la especificación del factor. Estos resultados son robustos a la presencia o ausencia de un activo sin riesgo.