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ON TAXATION IN A MODEL OF INTERGENERATIONAL ALTRUISM

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This paper analyzes the effects of taxation in a model of intergenerational altruism characterized by one representative individual à la Barro, with the focus on the impact of different taxes on steady-state equilibria. The paper also deals with some aspects of the interaction between non-lump-sum taxes and public debt issue in the discussion of the so-called «Ricardian equivalence theorem», i.e., the proposition that public debt and taxes have the same effects on the economy.

1. Introduction

Since the seminal contribution by Barro (1974) and the restatement of the so-called «Ricardian equivalence theorem», i.e., the proposition that public debt and taxes have the same effects on the economy, his approach of intergenerational altruism has become, although not a generally accepted view, at least a respectable one. In his model, the intergenerational linkages derived from the dependence of the utility functions imply that individuals, in spite of having finite lives, behave as if they were infinitely lived. A consequence is that lump-sum redistribution across generations, such as debt financing or a pay-as-you-go social security, has no effect on capital accumulation and factor returns.

The renewed interest about the equivalence of debt and taxes has given rise to a body of literature, with a series of equivalence theorems and propositions [Drazen (1978), Buiter (1979, 1980), Carmichael (1982), Burbidge (1983)], different ways to approach the model [Buiter and Carmichael (1984), Burbidge (1984), Abel (1987), Hillier and Lunati (1987), Bernheim and Bagwell (1988), López-García (1989)], and the discussion of the connection with pure life-cycle models [Weil (1987), López-García (1990)]. As for the effects of distortionary taxation, Chamley (1986) has analyzed the optimal tax treatment of capital income in economies in which agents have infinite lives,

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and Kotlikoff (1984, 1989.a) has dealt with some questions related to non-lump-sum taxes[1].

This paper discusses the effects of taxation in a model of intergenerational altruism characterized by one representative individual à la Barro (1974). Only one-sided altruism is considered, and the focus is on the impact of different taxes on steady-state equilibria[2]. Although some results are not new, it provides a simple and unified framework in which to consider these effects as well as their relationship with the kind of intergenerational concern underlying the model. Furthermore, the paper is related to some aspects of the «Ricardian equivalence theorem», i.e., the proposition that shifts between debt and tax finance for a given amount of public expenditure have no effects on the economy. It is well known that one of the assumptions required for the theorem to hold is that deficits are financed through lump-sum taxes [Barro (1978), Tobin and Buiter (1980), Brennan and Buchanan (1980)]. While Barro (1979) has himself developed a model in which taxes entail collection costs (and in which public debt becomes a mechanism for «tax smoothing»), there does not seem to be an analysis of the interaction of public debt and some non-lump-sum taxes in the standard model of intergenerational altruism. However, taxes in the real world are non-lump-sum, and this source of non-neutrality is not less important for being so obvious. The variety of non-lump-sum taxes could well affect the individuals' intertemporal budget constraints in ways that they would not be able to neutralize by means of offsetting changes in their behaviour.

Section 2 presents the basic model and discusses the implications of the type of voluntary intergenerational transfer, i.e., bequests from parents to children or gifts from children to parents, being operative. Section 3 characterizes steady-state equilibria in both cases and derives the results associated with different tax bases. Section 4 offers some concluding comments.

2. A model of intergenerational altruism

The basic model is that of overlapping generations developed in its pure life-cycle version by Samuelson (1958) and Diamond (1965), and extended by Barro (1974) to include intergenerational altruism and voluntary intergenerational transfers. The technology is described by a constant-returns-to-scale production function whose arguments are capital and labour. Therefore, output per unit of labour in period t, y_t, can be written as $y_t=f(k_t)$, $f' > 0$, $f'' < 0$, where k_t is capital per unit of labour in period t. If each factor is paid its marginal product:

$$r_t=f'(k_t) \qquad\qquad w_t=f(k_t)-k_t f'(k_t) \qquad [1]$$

[1] A related contribution is that of Calvo, Kotlikoff and Rodríguez (1979) in their comment on Feldstein's (1977) paper about the incidence of a tax on land rent.

[2] Two-sided altruism is dealt with in Buiter (1980), Burbidge (1983), Kimball (1987), Abel (1987) and Visaggio (1991).

where r_t and w_t are the gross of tax interest rate and wage rate respectively. The demographic structure is the simplest one, i.e., population is composed of individuals who live for two periods. In the first one they work and supply an amount of labour of their choice, while in the second one they fully retire. If L_t is the number of workers in period t, we have $L_t = (1+n)L_{t-1}$, where n is the population growth rate.

The government finances its consumption expenditure, E_t in period t, with the revenues provided by taxation and the issue of public debt, D_t being the debt (real-valued bonds) issued in period $t-1$ and to be repaid at the interest rate r_t in period t. The government's budget constraint is therefore $E_t + r_t D_t = T_t + (D_{t+1} - D_t)$, where T_t stands for tax revenue and $(D_{t+1} - D_t)$ is the variation in the stock of public debt in period t. Taxes are represented by means of a proportional tax rate, τ_i, on each of the i tax bases, i being c (consumption), w (wage income), r (capital income), b (bequests) and g (gifts)[3]. Hence, the government's budget constraint in period t can be written in per worker terms as:

$$e_t l_t + r_t(1-\tau_r)d_t l_t = \qquad [2]$$

$$[(1+n)d_{t+1}l_{t+1} - d_t l_t] + \tau_c\left[c_t^1 + \frac{c_{t-1}^2}{1+n}\right] + \tau_w w_t l_t + \tau_r r_t k_t l_t + \frac{\tau_b b_{t-1}}{1+n} + \tau_g g_t$$

where $e_t = E_t/L_t l_t$, $d_t = D_t/L_t l_t$ and $k_t = K_t/L_t l_t$ i.e., variables expressed in terms of units of labour, c_t^1 and l_t denote the first-period consumption and the labour supplied by an individual born in period t, c_{t-1}^2 the second-period consumption by an individual born in period $t-1$, b_{t-1} the bequest left in (his second) period t by an individual born in period $t-1$, and g_t the gift made by an individual in (his first) period t. Notice also that $(1+n)d_{t+1}l_{t+1} - d_t l_t = (D_{t+1} - D_t)/L_t$, i.e., the variation in the stock of public debt in period t in per worker terms, and that capital taxes are paid on debt interest.

Altruism from parents to children, i.e., a situation in which individuals derive utility not only from their own consumption but also from the welfare enjoyed by their descendants, this giving rise to the possibility of bequests, can be modelled, following Barro (1974), as $U_t = u(c_t^1, c_t^2, l_t, U_{t+1}^*)$, where U_{t+1}^* is the maximum utility level attainable by a representative descendant given the bequest received. The recursive dependence of the utility function entails that individuals, in spite of having finite lives, behave as if they expected to live forever. On the other hand, altruism from children to parents, i.e., a situation in which individuals derive utility from the welfare enjoyed by their ancestors, this generating gifts from the younger to the older generation, can be modelled, again in terms of Barro (1974), as $U_t = u(c_t^1, c_t^2, l_t, U_{t-1}^*)$, where U_{t-1}^* denotes the maximum utility level attainable by a representative ancestor given the gift received. In this case, individuals behave as if they had been living forever.

[3] An income tax, τ_y, can be viewed as an equal rate tax on wage income and capital income.

We shall only analyze one-sided altruism and, in order to find concrete results, we shall assume the additively separable utility function which has been extensively used [Buiter (1979, 1980), Carmichael (1982), Burbidge (1983)]. Thus, both kinds of altruism can be collapsed into the following expression:

$$U_t = v(c_t^1, c_t^2, l_t) + hU_{t+1}^* + jU_{t-1}^* \qquad [3]$$

where $v(.)$ has the standard properties and h and j are discount factors, with $j=0$ and $0 < h < 1$ ($h=0$ and $0 < j < 1$) for the case of altruism from parents to children (from children to parents)[4].

In the case of bequests, the individual budget constraint can be written as the equality between the present value of lifetime consumption (allowing for taxes) plus the total bequest, b_t, to be left, and the present value of lifetime resources, that is, the sum of net labour income and the bequest received at the beginning of one's second period, $b_{t-1}(1-\tau_b)/(1+n)$. Bequests are shared equally among all descendants, and they are constrained to be non-negative, i.e. $b_t \geqslant 0$ for all t. Perfect foresight is assumed to prevail, and we make the standard Nash assumption that the bequest an individual receives (and the same applies to the gift) is treated as given. When the transfers adopt the form of gifts, the individual budget constraint becomes the equality between the present value of consumption (again allowing for taxes) plus the gift made, g_t, and the present value of lifetime resources, where, since the individual has $(1+n)$ children, the total gift received is $(1+n)\, g_{t+1}\, (1-\tau_g)$. Gifts cannot be negative, i.e. $g_t \geqslant 0$ for all t.

As with [3], we can consider both kinds of transfers just writing:

$$(1+\tau_c)\,c_t^1 + \frac{(1+\tau_c)\,c_t^2}{1+r_{t+1}(1-\tau_r)} + \frac{b_t}{1+r_{t+1}(1-\tau_r)} + g_t = \qquad [4]$$

$$w_t(1-\tau_w)\,l_t + \frac{b_{t-1}(1-\tau_b)}{1+n} + \frac{(1+n)g_{t+1}(1-\tau_g)}{1+r_{t+1}(1-\tau_r)}$$

with $g_t = 0$ ($b_t=0$) on the case of operative bequests (gifts).

[4] The choice of the discount factor was the subject of the exchange between Buiter and Carmichael (1984) and Burbidge (1984). In their formulation of altruism from parents to children, Buiter (1979, 1980) and Carmichael (1982) assume that $h = 1/(1+\delta)$, i.e., that a parent takes care of (and discounts at a rate δ) the welfare of a representative child. On the other hand, Burbidge (1983) models utility dependence by assuming that a parent cares about the total welfare of all his $(1+n)$ «children», i.e., $h = (1+n)/(1+\delta)$. As for altruism from children to parents, while Buiter (1979, 1980) and Carmichael (1982) assume that a child discounts (at a rate ρ) the welfare of a representative parent, i.e., $j=1/(1+\rho)$, Burbidge (1983) posits a reverse discounting of the total utility of the $1/(1+n)$ «parents», so that $j=(1+\rho)/(1+n)$. A synthesis view, arguing that the qualitative results are the same, is suggested in López-García (1989).

From the maximization problem of either of the two versions of [3] subject to [4] we obtain the standard equalities of the marginal rates of substitution between consumptions and labour supply with their «relative prices»:

$$\frac{\partial v(c_t^1, c_t^2, l_t)/\partial c_t^1}{\partial v(c_t^1, c_t^2, l_t)/\partial c_t^2} = 1 + r_{t+1}(1-\tau_r) \qquad [5]$$

$$\frac{\partial v(c_t^1, c_t^2, l_t)/\partial l}{\partial v(c_t^1, c_t^2, l_t)/\partial c_t^1} = \frac{-w_t(1-\tau_w)}{(1+\tau_c)} \qquad [6]$$

In the case of altruism from parents to children giving rise to operative bequests, we can obtain a relationship between the marginal utility of own second-period consumption and the marginal utility of bequests. From the first-order condition with respect to b_t we find $h(\partial U_{t+1}^*/\partial b_t) = \lambda_t(1 + r_{t+1}(1-\tau_r))$, so that:

$$\frac{h[1 + r_{t+2}(1-\tau_r)]\,(1-\tau_b)}{(1+n)} \frac{\partial v(c_{t+1}^1, c_{t+1}^2, l_{t+1})}{\partial c_{t+1}^2} = \frac{\partial v(c_t^1, c_t^2, l_t)}{\partial c_t^2} \qquad [7]$$

where use has been made of the fact that $\partial U_t/\partial b_{t-1} = \lambda_t(1-\tau_b)/(1+n)$, and λ_t is the Lagrange multiplier associated with [4].

As for altruism from children to parents, assuming an interior solution for gifts, [7] is replaced by a relationship between the marginal utility of own first-period consumption and the marginal utility of gifts. Since the first-order condition with respect to g_t becomes $j(\partial U_{t-1}^*/\partial g_t) = \mu_t$, we can write:

$$\frac{j(1+n)(1-\tau_g)}{1 + r_t(1-\tau_r)} \frac{\partial v(c_{t-1}^1, c_{t-1}^2, l_{t-1})}{\partial c_{t-1}^1} = \frac{\partial v(c_t^1, c_t^2, l_t)}{\partial c_t^1} \qquad [8]$$

where $\partial U_t/\partial g_{t+1} = \mu_t(1+n)(1-\tau_g)/[1 + r_{t+1}(1-\tau_r)]$, and μ_t is the Lagrange multiplier.

The equilibrium in the capital market requires that the stock of capital plus debt at the beginning of period $t+1$ be equal to the sum of the savings of the L_t members of generation t during their first period of life and, if bequests are operative, of the transfers made by the L_{t-1} members of generation $t-1$ at the end of their lives:

$$w_t(1-\tau_w)\,l_t - g_t - (1+\tau_c)c_t^1 + \frac{b_{t-1}(1-\tau_b)}{(1+n)} = (1+n)\,[k_{t+1} + d_{t+1}]\,l_{t+1} \qquad [9]$$

Combining [1], [2], the relevant version of [4], and [9], we obtain:

$$y_t\, l_t = c_t^1 + \frac{c_{t-1}^2}{(1+n)} + e_t l_t + (1+n)\, k_{t+1}\, l_{t+1} - k_t l_t \qquad [10]$$

i.e., the aggregate feasibility condition.

3. Steady-state equilibria

A steady-state equilibrium in this model will be a situation in which the capital-labour ratio, and along with it factor returns, consumption levels and voluntary transfers, do not change. Assuming such a steady state to exist, we can characterize long-run equilibria by just deleting the time subscripts in [7] and [8].

Using [7] it is direct that the following expression must be verified in a steady state with operative bequests:

$$[1+r(1-\tau_r)](1-\tau_b)=\frac{1+n}{h} \qquad [11]$$

from which it can be seen that a steady state with positive bequests will entail $r(1-\tau_r) > n$, i.e., a net of taxes rate of return to capital *greater* than the population growth rate. The steady-state equilibria in this case of altruism from parents to children can be characterized by means of the system of seven equations represented by the steady-state versions of [1], [2], [4], [5] or [6], and [9], which can be relabelled as [1'], [2'], etc., in addition to [11]. In particular, [2] becomes:

$$el+[r(1-\tau_r)-n]\,dl=\tau_c\left[c^1+\frac{c^2}{(1+n)}\right]+\left[\tau_w wl+\tau_r rkl+\tau_b\frac{b}{(1+n)}+\tau_g g\right] \qquad [2']$$

and given the values of the fiscal parameters τ_i, d and e, there are seven unknowns, c^1, c^2, l, k, r, w and b. The sequential description of the solution of the model is as follows. Equation [11] determines r, which in turn allows to find k and w in the factor payment conditions [1']. Then, c^1, c^2 and l are obtained with [5'], [6'], and the steady-state feasibility condition:

$$c^1+\frac{c^2}{(1+n)}+el=[f(k)-nk]\,l \qquad [10']$$

Finally, the value of b is found using [4'].

The same reasoning can be applied to the case of gifts. In effect, the steady-state equilibria can be seen from [8] that fulfill the following expression:

$$\frac{1+r(1-\tau_r)}{(1+\tau_g)}=j(1+n) \qquad [12]$$

from which it follows that $r(1-\tau_r) < n$, i.e., a steady state with positive gifts will display a net of tax interest rate *less* than the rate of demographic growth. The long-run equilibria in the case of altruism from children to parents can be characterized in terms of the system of seven equations given by [1'], [2'], [4'], [5'], or [6'], and [9'], in addition to [12], where the unknowns are the same as in the model with bequests replacing b by g. The sequential solution also follows the same steps as in the preceeding analysis.

The interpretation of [11] and [12] in terms of the incidence of taxation is direct. Since τ_c and τ_w do not appear in these expressions, neither consumption taxes nor wage income taxes will have any effect on steady-state capital accumulation. Concerning capital income taxation, we can observe that increases in τ_r will increase the gross of tax interest rate and decrease the capital-labour ratio. The same can be said about taxes on bequests, since the greater τ_b the greater r in [11]. However, gifts taxation contributes to capital accumulation, since the greater τ_g, the smaller r in [12].

As for the effects of public debt and its potential neutrality, it is well-known that in this framework, if taxes are lump-sum, the «Ricardian equivalence theorem» holds. In effect, the steady-state values of the interest rate and the capital per unit of labour ratio are not modified by the public debt issue, i.e., the changes in d are neutral in the steady state [Barro (1974), Carmichael (1982)]. However, the result that the steady-state values of r and k are not affected by the public debt also holds when deficits are financed with wage and/or consumption taxation. This follows from the fact that neiter τ_c nor τ_w appear in [11] and [12]. The reason is that, regardless of the direction of the transfers, this model of intergenerational altruism implies a long-run supply of saving perfectly elastic at the interest rates implicitly given in [11] and [12], which do not depend on τ_c or τ_w.

This point also illustrates why the result of invariance of k with respect to the value of d fails to hold in the presence of capital income taxation or with inheritance or gift taxation. In this case the long-run interest elasticity of the supply of saving continues to be infinite, but the relevant interest rates do depend on τ_r, τ_b and τ_g. Notice, in any case, that these effects are not attributable to the debt *per se*, but to the kind of distortionary taxes used.

4. Concluding comments

This paper has discussed the impact of different taxes on long-run equilibria in a model of intergenerational altruism characterized by one «representative individual» à la Barro (1974). Some insights have also been obtained about the effects of relaxing the assumption of lump-sum taxes in the discussion of the «Ricardian equivalence theorem». However, two points must be emphasized. The first one is that the analysis has focused on steady states, thus neglecting the questions related to the transitions between long-run equilibria. This means that even in the case in which two taxes are equivalent when comparing steady-state configurations, their effects may well be very different along the *time path* of convergence to the steady state. The second one is that when individuals are heterogeneous, family linkages form complex networks in which each individual may belong to many dynastic groupings, this proliferation of linkages between families giving rise to the result that *everything* is neutral [Bernheim and Bagwell (1988), Kotlikoff (1989b)]. Actually, the very question of whether a model with voluntary intergenerational transfers or the more tradicional life-cycle theory is the most relevant in explaining savings remains a controversial question [Modigliani (1988), Kotlikoff (1988)]. It seems fair to say that more research on this topic is warranted.

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Resumen

En este trabajo se analizan los efectos de distintas formas de imposición en el marco de un modelo de altruismo intergeneracional caracterizado por un solo individuo representativo à la Barro, poniendo el énfasis en el impacto de las diversas figuras tributarias en los estados estacionarios. Se tratan también algunos aspectos de la interacción entre unos impuestos que no son de suma fija y la emisión de deuda pública en la discusión del «teorema ricardiano de la equivalencia», es decir, la proposición de que la deuda pública y los impuestos dan lugar a los mismos efectos.

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