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COMPETITION ENHANCING MEASURES AND SCOPE ECONOMIES: A WELFARE APPRAISAL

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This paper examines welfare changes before the introduction of more competition in technologically related markets. We develop a simple two-market duopoly with product differentiation where a multi-product firm competes with a different single-product firm in each market. Two competition enhancing measures are considered, divestiture of the multi-product firm and entry of single-product firms in one of the markets. The results obtained indicate that more competition may lead to a welfare reduction. Our analysis points out the relevance of the type and size of economies of scope, the particular way of introducing more competition and the degree of product differentiation.

Keywords: Cost complementarity/substitutability, fixed cost subadditivity, welfare.

(*JEL* L13, L59)

1. Introduction

Firms are typically faced with the decision whether to adopt a flexible technology, which allows them to produce a range of products. One of the reasons for multi-production lies in the existence of scope economies.[1] This view emphasizes the role of technology in the determination of firm and industry structure. Then an important source of potential competition is acknowledged to come from firms already producing in markets that are effectively connected on cost grounds. Consequently, regulatory measures aimed at increasing competition in

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[1] The existence of economies of scope is connected to the presence of inputs that may be effectively shared among production processes, “quasi-public” inputs and other related factors, such as managerial expertise, a good financial rating or a sales staff. See Panzar (1989) and Milgrom and Roberts (1990).

one particular market must be carefully looked at since they can affect competition in another market. Such an "across markets" effect occurs through the output reallocation by multi-product firms. This paper examines how competition intensity and social welfare vary when enhancing competition measures are adopted in technologically related markets. Our analysis points out the relevance of the type and size of economies of scope, the particular way of introducing more competition and the degree of product differentiation.

There are quite a number of studies devoted to the investigation of competition intensity in multi-market oligopoly that concentrate on demand-side linkages (see e.g. Brander and Eaton, 1984, Bernheim and Whinston, 1990, and Shaked and Sutton, 1990). Externalities across markets may also happen on cost-side linkages. One strand of the literature has studied the strategic choice between a flexible (diversified) technology and a dedicated (specialized) technology in cost-related markets, as done by Röller and Tombak (1990), Eaton and Lemche (1991) and Dixon (1994). Also related is Calem (1988) who examines entry and entry deterrence into a market occupied by a monopolist. A multi-product cost function may exhibit economies of scope either through the fixed or the variable costs components. Röller and Tombak (1990), in a differentiated products setting, assume that the choice of either technology has no implication for marginal production costs whereas Dixon (1994), in a homogeneous products setting, assumes the opposite. Only the latter case translates to firms' reaction functions. Then, the sign of the cross-derivatives of the cost function[2] becomes crucial in assessing any welfare gains in the presence of increased competition.

Rather often, competition authorities question the desirability of having large multi-product firms in some industries. The above basic reasoning precisely indicates the sensitivity of the conclusions to the properties of the cost function. Thus, the move from a multi-product duopoly to a two-monopoly structure can lead to welfare gains when there is cost substitutability, as in Dixon (1994), and yet to welfare losses when technology choice only affects fixed costs, as in Röller and Tombak (1990). Further, the celebrated paper by Bulow et al. (1985)

[2] Consider the two-product example. A multi-product cost function $C(q_1, q_2)$ exhibits cost complementarity if $\partial^2 C/\partial q_1 \partial q_2 < 0$. It implies that the marginal cost of producing one product decreases with increases in the quantity of the other product. The cost function exhibits cost substitutability when $\partial^2 C/\partial q_1 \partial q_2 > 0$.

includes an example of a multi-product firm that faces competition in one market and is a price-taker in the other. Interestingly enough, a lower price results in lower profits to the multi-product firm and this is because its cost function exhibits diseconomies of scope.

The present paper is motivated by and complements these three analyses. To this end, we will begin by setting out a two-market duopoly where a multi-product firm competes with a different single-product firm in each market. This reference model will be referred to as the *diversified regime*. The analysis will distinguish economies of scope stemming from the fixed and from the variable costs components.[3] The degree of product differentiation also plays an important role when we depart from the reference model and introduce two competition enhancing measures. Firstly, we will suppose that the multi-product firm is split into two single-product firms, which we designate as the *corporate divestiture regime*. Secondly, we will assume entry of single-product firms in one of the markets. This is in fact equivalent (in the limit) to assuming one of the single-product firms behaving competitively. This will be called the *competitive regime*.

Some examples may be useful in placing the paper into perspective. A considerable amount of empirical research on the assessment of economies of scale in joint production has been devoted to the transport and the telecommunications industries. Market structure in these industries typically consists of one or several multi-product firms competing against different single-product firms in several markets. Thus, railway companies supply both long-distance (alternatively passenger) services and urban (alternatively freight) services. They face competition from other transport modes in different markets. Great Britain and Sweden have pioneered a very strong divestiture process of British Rail and Statens Jarnvager, respectively. In telecommunications, domestic state-owned telephone companies initially provided basic telephony services. Later, these companies have usually been awarded a license to operate in the cellular telephone market. Gradual liberaliza-

[3] Economies of scope are present whenever the cost of producing both outputs together is less than the total cost of producing each product in a separate firm, that is, $C(q_1, q_2) < C(q_1, 0) + C(0, q_2)$. For the two-product case this is equivalent to saying that the cost function is (strictly) subadditive. The existence of cost complementarity implies the existence of economies of scope. Also, economies of scope may occur even in cases in which there is cost substitutability as long as the cost function is sufficiently subadditive in fixed costs.

tion in the form of entry of new operators has brought competition by firms which are not present in both the basic and mobile telephony.[4]

The analysis shows that modelling an increase in competition as a move from the diversified regime to either the corporate divestiture regime or the competitive regime leads to ambiguous welfare variations. We offer a taxonomy in terms of the type and the size of economies of scope, and the way of introducing more competition. More specifically, the move to the corporate divestiture regime entails welfare losses when there is cost complementarity. The main reason for it is that economies of scope cannot be exploited and an inefficiency in production is generated. However, welfare may increase under cost substitutability and fixed cost subadditivity. Splitting up the multi-product firm results in greater output by both the resulting single-product firms. In other words, keeping a diversified firm causes the inefficiency in production. Values of cost substitutability above a critical level and a low degree of product differentiation are conditions which ensure that a move to corporate divestiture is welfare improving. By contrast, the move to the competitive regime may result in a welfare reduction in the presence of cost complementarity whereas welfare gains arise as long as there is cost substitutability. Entry generates an output shift from the multi-product to the single-product firms in both markets. With cost substitutability and fixed cost subadditivity, entry produces an equilibrium output mix by the multi-product firm such that total output raises in both markets resulting in an unambiguous welfare increase. However, with cost complementarity, the gain associated with

[4] See e.g. Kessides and Willig (1995) and Campos-Méndez and Cantos-Sánchez (2000) for evidence on the existence and significance of cost complementarities across different rail operations. Banker *et al.* (1998) provide evidence on scope economies in the U.S. telecommunications industry. The Swedish rail company, Statens Jarnvager, experienced the first vertical separation process in Europe. British Rail has undergone a very strong separation process at two levels: a) infrastructure and operations and, b) operations themselves into twenty-five groups of routes (TOCS, Train Operating Companies). Furthermore, the rail freight sector, initially divided into three separate companies, now remains almost a monopoly (see e.g. Nash, 1997, and Campos-Méndez and Cantos-Sánchez, 2000). Typically, in the European telecommunications sector, more competition has been firstly introduced in mobile telephony and subsequently in basic telephony. There has been both entry of new operators and the creation of some regulatory body - such as OFTEL in the UK and ART in France. The fares proposed to users are regulated by government and national authorities. Waverman and Sirel's (1997) survey exemplifies the asymmetries caused by some unexpected effects of deregulatory measures in view of enhancing competition.

more competition offsets the productive efficiency loss, and hence results in a welfare gain, only for low values of cost complementarity and the higher the degree of product differentiation.

We begin by describing the model. Section 3 compares the move from the diversified to the corporate divestiture regime. The move to the competitive regime is taken up in Section 4. A Table summarizes the welfare taxonomy and some industries worth of analysis are suggested. Some concluding remarks close the paper.

2. Description of the model

There are two products, A and B, offered in markets A and B, respectively. Two varieties of each product are present in each market, one produced by a multi-product firm, denoted by subscript M, and one produced by a single-product firm, denoted by subscript S. This market structure can be justified on the following grounds. On the one hand, there may be regulatory constraints at play as suggested by the examples above.[5] On the other, such a market structure might emerge endogenously as the equilibrium choice of a two-stage game where firms choose their production technology and then compete in the market. More specifically, suppose, as in Röller and Tombak (1990) and Dixon (1994), that in the first stage (two symmetric) firms simultaneously choose between a flexible technology, and become multi-product, and a dedicated technology, and become single-product. There is a Cournot game in the second stage. Assume that the cost of purchasing the flexible technology is T and that, for simplicity, the dedicated technology is free. Then it can be shown that an asymmetric structure, where one firm chooses a flexible technology and the other does not, is obtained for a certain range of T values.[6]

[5] Waverman and Sirel (1997) write: "...U.K. regulators have encouraged cable TV companies to provide phone service, but *prohibited* British Telecom from entering the television business." (emphasis added). Similarly, Spanish Telefonica was not allowed to compete with cable operators for a given period of time.

[6] Denote by $\Pi(fl, fl)$ the variable profits to a firm when both firms choose flexible technologies. Similarly, let $\Pi(de, de)$ denote the variable profits when both firms choose dedicated technologies. Finally, $\Pi(fl, de)$ and $\Pi(de, fl)$ denote the asymmetric cases. The pair (fl, de) will be a Nash equilibrium if i) $\Pi(fl, de) - T > \Pi(de, de)$, and ii) $\Pi(de, fl) > \Pi(fl, fl) - T$. These two conditions can be rewritten as $\Pi(fl, de) - \Pi(de, de) > T > \Pi(fl, fl) - \Pi(de, fl)$. The asymmetric equilibrium will arise as long as the cost of purchasing the flexible technology, T, lies between the opportunity cost of adopting the flexible technology when none of the firms

There is then a duopoly structure in each market. Therefore we have the following (inverted) demand system for differentiated products:[7]

$$\begin{array}{l} p_{Mi} = D_i - b_i q_{Mi} - d q_{Si} \\ p_{Si} = D_i - b_i q_{Si} - d q_{Mi} \end{array} \qquad [1]$$

for $i = A, B$. It is assumed that $b_i > d > 0$, which means that the varieties of each product are substitutes and that the "own-price" effect dominates the "cross-price" effect. The ratio d^2/b_i^2 is a measure of the degree of differentiation. As it tends to one, varieties are more homogeneous. Thus, products A and B are not demand-related but the whole system is linked due to the existence of scope economies enjoyed by the multi-product firm. Formally, the cost function is said to enjoy economies of scope when $C(q_{MA}, q_{MB}) < C(q_{MA}, 0) + C(0, q_{MB})$. As noted in the Introduction, economies of scope may be caused by the variable cost or the fixed cost components. It will then be useful to distinguish two different constructions for the joint cost function:

$$\hat{C}_M = \left\{ \begin{array}{llllll} c_A q_{MA} & +c_B q_{MB} & -w q_{MA} q_{MB} & \text{for} & q_{MA} > 0 & q_{MB} > 0 \\ c_A q_{MA} & & & \text{for} & q_{MA} > 0 & q_{MB} = 0 \\ & c_B q_{MB} & & \text{for} & q_{MA} = 0 & q_{MB} > 0 \end{array} \right. \qquad [2]$$

$$\tilde{C}_M = \left\{ \begin{array}{lllllll} c_A q_{MA} & +c_B q_{MB} & +w q_{MA} q_{MB} & +F_{AB} & \text{for} & q_{MA} > 0 & q_{MB} > 0 \\ c_A q_{MA} & & & +F_A & \text{for} & q_{MA} > 0 & q_{MB} = 0 \\ & c_B q_{MB} & & +F_B & \text{for} & q_{MA} = 0 & q_{MB} > 0 \end{array} \right. \qquad [3]$$

Costs for the single-product firms are $C_i = c_i q_{Si}$, $i = A, B$. Parameters c_A and c_B denote the marginal cost of producing A and B separately. The joint cost function has got an "interactive term" meaning that costs are reduced (or increased) in proportion to the product of

do it, and the opportunity cost of adopting the flexible technology when the rival adopts it.

[7] This demand system follows from the maximization of a quasiconcave utility function of a representative consumer as follows: $U(q_{MA}, q_{SA}) = D_A q_{MA} + D_A q_{SA} - (b_A q_{MA}^2 + 2d q_{MA} q_{SA} + b_A q_{SA}^2)/2$, for market A; symmetrically for market B. See Singh and Vives (1984).

the outputs in each market. In particular, the cost function $\hat{C}_M$ exhibits cost complementarity, $\partial^2 C(q_{MA}, q_{MB})/\partial q_{MA}\partial q_{MB} = -w < 0$. As shown in Panzar (1989), the existence of weak cost complementarity implies the existence of economies of scope. Note that the marginal cost of production for each product must be non-negative, hence the value of w is bounded above (specifically, $w < \frac{c_A}{q_{MB}}$ and $w < \frac{c_B}{q_{MA}}$). On the other hand, the cost function $\tilde{C}_M$ exhibits cost substitutability, $\partial^2 C(q_{MA}, q_{MB})/\partial q_{MA}\partial q_{MB} = w > 0$. Fixed cost subadditivity means that $F_{AB} < F_A + F_B$. Thus, there will be economies of scope if $\tilde{C}_M$ is sufficiently subadditive in the fixed cost component. There is then an upper bound on w, i.e., $\tilde{C}_M$ exhibits economies of scope as long as $0 < w < (F_A + F_B - F_{AB})/q_{MA}q_{MB}$. Both [2] and [3] satisfy the regularity conditions that characterize a multi-product cost function, as established in Panzar (1989).

The joint cost function $\hat{C}_M$ has the following properties: a) the multi-product firm enjoys increasing economies of scale in the production of both outputs but constant product specific scale economies, b) each product's marginal cost and average incremental cost are independent of that product's level. On the other hand, $\tilde{C}_M$ satisfies: a') increasing economies of scale if $w < F_{AB}/q_{MA}q_{MB}$ and increasing product specific scale economies, and b') each product's average incremental cost is decreasing in q_{Mi}. These properties are proven in Appendix A2.

Suppose for a moment, and to highlight the role played by the cross-derivatives of the joint cost function, that there were economies of scope based only on fixed cost subadditivity. Then, the two markets would not be linked and any competition enhancing measure taken in one market would no longer affect competition intensity in the other market. In particular, the move to the corporate divestiture regime would unambiguously entail a welfare reduction. The output equilibrium choices are the same regardless that the multi-product firm is kept together or separated into two single-product firms. Since $F_{AB} < F_A + F_B$ it follows that welfare is larger under the diversified regime. Consider now the move to the competitive regime. Entry in one of the markets leads to an increase in aggregate output whereas nothing happens in the other market. As the number of firms increases the price-marginal cost gap shrinks and, when making the welfare comparison, fixed costs cancel out. Consequently, welfare is unambiguously larger under the competitive regime. Such conclusive predictions cannot be reached in the presence of scope economies coming from the variable costs components; the two markets are connected and any policy measure taken in one market is transmitted to the other market. This motivates the following analysis.

3. A move to the corporate divestiture regime

We begin by characterizing the diversified regime, i.e. the equilibrium previous to the introduction of more competition. Firms maximize profits by choosing quantities simultaneously and independently. Let $a_A = D_A - c_A$ and $a_B = D_B - c_B$. Hence, for the cost function $\hat{C}_M$, the problem is stated as

$$\max_{q_{MA},q_{MB}} \Pi_M = \sum_{i=A,B} (a_i - b_i q_{Mi} - d q_{Si}) q_{Mi} + w q_{MA} q_{MB} \qquad [4]$$

$$\max_{q_{SA}} \Pi_A = (a_A - b_A q_{SA} - d q_{MA}) q_{SA} \qquad [5]$$

$$\max_{q_{SB}} \Pi_B = (a_B - b_B q_{SB} - d q_{MB}) q_{SB} \qquad [6]$$

The two markets clear simultaneously. The Cournot equilibrium quantities, where superscript df stands for diversified, are the following:[8]

$$q_{Mi}^{df} = \frac{a_i[(4b_j^2 - d^2)(2b_i - d)] + 2a_j b_i (2b_j - d) w}{(4b_A^2 - d^2)(4b_B^2 - d^2) - 4b_A b_B w^2} \qquad [7]$$

$$q_{Si}^{df} = \frac{a_i[(4b_j^2 - d^2)(2b_i - d) - 2b_j w^2] - a_j d (2b_j - d) w}{(4b_A^2 - d^2)(4b_B^2 - d^2) - 4b_A b_B w^2} \qquad [8]$$

for $i, j = A, B$, $i \neq j$. The parameter w positively affects both multi-product firm's outputs whereas they reduce the equilibrium quantities of the single-product competing firms. In output space, as compared with the case without economies of scope, cost complementarity implies that firm M's reaction function shifts outwards and this leads to a reduction in the output of the single-product competitor since

[8] Note that three technical requirements must be satisfied: a) second-order and stability conditions, b) interior solutions in quantities, and c) either positive marginal costs under cost complementarity, or sufficient fixed cost subadditivity under cost substitutability. In the analysis the most restrictive bounds are used. See Appendix A3.

outputs are strategic substitutes. The presence of the interactive term makes this happen in both markets. Profits of the multi-product firm increase at the expense of a decrease in profits of the single-product firms. The analysis for the cost function $\tilde{C}_M$ is straightforward, where now the parameter w enters negatively in [7] and positively in [8] in the linear term of the numerators.

The corresponding equilibrium quantities under the corporate divestiture regime, superscript dv, follow by taking $w = 0$ in [7] and [8], which yield the standard differentiated Cournot outcome in each market $q_{Mi}^{dv} = q_{Si}^{dv} = \frac{a_i}{2b_i+d}$. We may then compare total welfare (producers profits plus consumer surplus) under both regimes.

PROPOSITION 1 *A move from the diversified regime to the corporate divestiture regime is always welfare reducing for* $\widehat{C}$, *that is, with cost complementarities.*

PROOF: See Appendix A1.

Economies of scope are a central reason for the existence of multi-product firms. It is not surprising that divestiture leads to a welfare reduction when there are cost complementarities; they reflect that producing more of one product lowers the cost of manufacturing a second product. If the multi-product firm is separated then firm M's reaction function shifts inwards. This generates an inefficiency in the output allocation which in fact will result in smaller aggregate output in both markets. Thus, consumer surplus and aggregate profits are higher when cost complementarities can be exploited. Interestingly enough the move to the divestiture regime might be welfare improving although economies of scope were wasted. With cost substitutability and fixed cost subadditivity, the separation of the multi-product firm implies, in output space, a greater output by both the resulting single-product firms. Put differently, keeping it together originates an inefficiency in production since marginal costs are larger than the competitors'. Aggregate output in both markets is larger and this lends a hand for a welfare increase.

At this point our objective is not a search for generality, but rather to illustrate both the conditions and the manner by which, with cost substitutability and fixed cost subadditivity, more competition may be welfare improving. In doing so we assume symmetric markets, that is $a_A = a_B = a$ and $b_A = b_B = 1$. The social welfare difference has a variable (in terms of w) and a fixed cost component. For any given $d \in [0, 1]$, there is a value w^*, defined for each value of the ratio $\Delta F/a^2$, that sets that difference to zero. $\Delta F = F_A + F_B - F_{AB}$ is the fixed cost difference between separating the multi-product firm and keeping it together; a is a measure of oligopoly profitability. However, the direction of the welfare change does not solely depend on whether w lies above or below w^*. There is an upper bound on w for $\tilde{C}_M$ to exhibit scope economies. That bound, $\bar{w}$, is also a function of $\frac{\Delta F}{a^2}$. Further, there is an upper bound on the size of w imposed by the positive outputs and the second order conditions. These conditions impose that $w < 2 - \frac{d^2}{2}$. We can then state the following result.

PROPOSITION 2 *A move from the diversified regime to the corporate divestiture regime with cost substitutability and fixed cost subadditivity is welfare improving under the following conditions:*

i) If either $0 < \frac{\Delta F}{a^2} \leq \frac{2-d}{8(2+d)}$ and $w \in [w^*(\frac{\Delta F}{a^2}), \overline{w}(\frac{\Delta F}{a^2})]$

ii) or if $\frac{2-d}{8(2+d)} < \frac{\Delta F}{a^2} \leq \frac{28-8d-9d^2}{16(2+d)^2}$ and $w \in [w^*(\frac{\Delta F}{a^2}), 2 - (\frac{d^2}{2})]$.

PROOF: See Appendix A1.

Divestiture of the multi-product firm will lead to a welfare increase for large enough values of w and a certain size of the fixed cost difference relative to oligopoly profitability. It would not be surprising, as in Dixon (1994), to obtain a welfare gain when there are diseconomies of scope; it is better having two single-product firms rather than two multi-product firms. In contrast, we assume economies of scope and establish, for a class of examples, when it is socially preferable to have a duopoly in each market.

The intuition is as follows. Divestiture generates an output effect since the ensuing single-product firms increase their output. Such an output effect is more important the larger w is. Also, divestiture implies that average cost (of the resulting single-product firms) is higher as compared with (earlier) average incremental cost. Note that there are increasing returns under both regimes. Therefore, it must be the case that average cost at the corporate divestiture equilibrium be below average incremental cost at the diversified equilibrium in order to have an efficiency gain in production. When w exceeds w^* the average cost curves are closer to each other and the output effect is so significant that it compensates for the loss associated with not exploiting fixed cost subadditivity. Aggregate output in both markets increase which implies a gain for consumers. Thus, the efficiency gain in production and greater consumer surplus outweigh the profits loss by the competing single-product firms resulting in a welfare improvement. The output effect, and consequently the efficiency gain from divestiture, increases with the degree of product differentiation. Since the variables are strategic substitutes, the output reduction on the side of the single-product competitors is smaller the more differentiated products are. Consumer surplus depends on aggregate output and it is larger as d tends to one. Let us finally remark that if the cost function $\tilde{C}_M$ is strongly subadditive in fixed costs, then the output effect will never produce an efficiency gain. In fact, we can determine a sufficient condition ensuring that separating the multi-product firm always entails a welfare loss.

RESULT 1 *For all values of the ratio* $(\Delta F/a^2) \geq \frac{7}{16}$ *divestiture is always welfare reducing, regardless of the degree of product differentiation and the size of cost substitutability.*

PROOF: See Appendix A1.

4. A move to the competitive regime

Now suppose that entry of single-product firms is allowed in market B to analyze whether introducing more competition in this way has a positive effect on welfare. Assume there are n single-product (symmetric) firms in market B. Solving the profit maximization problem (of $n+2$ firms) yields the following equilibrium quantities $(n+3)$ when the multi-product firm's cost function is $\hat{C}_M$,

$$q^e_{MA} = \frac{a_A[(2b_A - d)(2(1+n)b_B^2 - nd^2)]+}{(4b_A^2 - d^2)(2(1+n)b_B^2 - nd^2) - 2(1+n)b_Ab_Bw^2}$$

$$\frac{2a_Bb_A((1+n)b_B - nd)w}{(4b_A^2 - d^2)(2(1+n)b_B^2 - nd^2) - 2(1+n)b_Ab_Bw^2} \qquad [9]$$

$$q^e_{MA} = \frac{a_B[(4b_A^2 - d^2)((1+n)b_B - d)]+}{(4b_A^2 - d^2)(2(1+n)b_B^2 - nd^2) - 2(1+n)b_Ab_Bw^2}$$

$$\frac{a_Ab_B(1+n)(2b_A - d)w}{(4b_A^2 - d^2)(2(1+n)b_B^2 - nd^2) - 2(1+n)b_Ab_Bw^2} \qquad [10]$$

$$q^e_{SA} = \frac{a_A[(2b_A - d)(2(1+n)b_B^2 - nd^2) - (1+n)b_Bw^2]}{(4b_A^2 - d^2)(2(1+n)b_B^2 - nd^2) - 2(1+n)b_Ab_Bw^2}$$

$$\frac{-a_B d((1+n)b_B - nd)w}{(4b_A^2 - d^2)(2(1+n)b_B^2 - nd^2) - 2(1+n)b_A b_B w^2} \quad [11]$$

$$q_{SB}^e = \frac{a_B[(4b_A^2 - d^2)(2b_B - d) - 2b_A w^2]}{(4b_A^2 - d^2)(2(1+n)b_B^2 - nd^2) - 2(1+n)b_A b_B w^2}$$

$$\frac{-a_A d(2b_A - d)w}{(4b_A^2 - d^2)(2(1+n)b_B^2 - nd^2) - 2(1+n)b_A b_B w^2} \quad [12]$$

where superscript e stands for entry. Note that there are n single-product firms producing q_{SB}^e. Let $Q_{SB}^e = nq_{SB}^e$. Equilibrium outputs change with n as follows: $\frac{\partial q_{MA}^e}{\partial n} < 0$, $\frac{\partial q_{MB}^e}{\partial n} < 0$, $\frac{\partial q_{SA}^e}{\partial n} > 0$, and $\frac{\partial Q_{SB}^e}{\partial n} > 0$. The corresponding equilibrium quantities for the cost function $\tilde{C}_M$ follow easily; the parameter w enters negatively in the numerator of [9]-[10] and positively in the linear term of the numerator of [11]-[12]. The limit of [9] to [12] as n tends to infinity results in equilibrium quantities that coincide with those obtained when the single-product firm in market B behaves competitively, i.e. it sets price equal to marginal cost, and the remaining firms behave strategically.

As the number of firms in market B increases the reaction function of the aggregate output of the single-product firms rotates outwards on the q_{MB}-axis. Since outputs are strategic substitutes, the multi-product firm reduces production at the expense of an increase in the aggregate output of single-product firms. These effects occur regardless that the cost function exhibits cost complementarity or cost substitutability, and their magnitude depends on the degree of product differentiation. In fact, total output in market B is larger than under the diversified regime. But entry in one market also affects equilibrium outputs in the other market. It takes place through the multi-product firm's reaction function in A, which shifts inwards in the presence of cost complementarity. As outputs are strategic substitutes, the multi-product firm in A reduces production. The magnitude of that reduction (in absolute terms) is larger the lower the degree of product differentiation. Furthermore, the decrease in q_{MA} has a feedback effect on market B for the marginal cost of producing q_{MB} is larger thus reinforcing the initial decrease in output; the feedback effect increases (in absolute terms) with the size of w. The effects in market A are exactly the opposite in the presence of cost substitutability, i.e. q_{MA}

increases whereas q_{SA} falls. The net effect of entry on welfare will crucially depend on the final output mix by the multi-product firm and the output reallocation within each of the markets. As stated in the next proposition, more competition might lead to a welfare reduction. It characterizes a class of examples from our general presentation under the assumption of identical degree of product differentiation in both markets.

PROPOSITION 3 *Suppose* $b_A = b_B = 1$. *The move from the diversified regime to the competitive regime involves a welfare improvement for* $\widetilde{C}_M$, *that is, with cost substitutability and fixed cost subadditivity. This is not necessarily true for* $\widehat{C}_M$, *i.e. in the presence of cost complementarity.*

PROOF: See Appendix A1.

The strategy of the proof consists of writing social welfare in the competitive regime and show that it is increasing with n. Under cost complementarity, entry can be catalogued as inefficient in the sense that, in both markets, there is an output shift from the multi-product firm to the single-product firms, whose marginal cost is higher. Total output in market B increases with entry and yet it falls in market A. As the numerical simulation in Table 1 shows, the productive efficiency loss is larger than the gain associated with more competition for large values of w and the lower the degree of product differentiation. The above mentioned output variations are stronger under these conditions. The Table displays w^m, which is the most restrictive upper bound from the technical requirements on w, and $\hat{w}$ which is the value of w such that $SW^e - SW^{df} = 0$, for different values of market size, D_i, costs, c_i, and $d \in [0, 1]$ for $i = A, B$. Entry entails a welfare reduction as long as, for each column, $w \in [\hat{w}, w^m]$. The value of $\hat{w}$ has been computed for two cases: when n equals two and when n tends to infinity. A dark shadowed cell identifies that a welfare reduction may happen in both cases; a light shadowed cell identifies that such a possibility exists only when n equals two. Finally, entry is always welfare enhancing in the remainder of the cells. For example, consider the row $c_A = c_B = 10$ in part 1.a of the Table. If $d = 0{,}8$ and the actual w is 1, then there is a welfare reduction in both cases, while if $d = 0{,}2$ entry always leads to a welfare increase. Note that as n increases the w interval for a welfare loss shrinks. The degree of product differentiation is higher as we move rightwards. Finally, the bottom part in the Table indicates that there may be a welfare production irrespective of entry taking place in the most profitable market.

TABLE 1
A move to the competitive regime:
Numerical simulations for the cost complementarity case

1.a. Symmetric markets

$a_B = a_A = a$ (D_A = 20, D_B = 20)

		d = 1	d = 0.8	d = 0.6	d = 0.4	d = 0.2
c_A=c_B=15	w^m	1	1.2	1.4	1.515	1.523
c_A=c_B=12.5	w^m	1	1.2	1.281	1.297	1.285
c_A=c_B=10	w^m	1	1.05	1.07	1.06	1.04
	$\hat{w}$ ($n \to \infty$)	0.790	0.967	1.158	1.365	1.598
	$\hat{w}$ (n=2)	0.704	0.895	1.096	1.313	1.559

1.b. Entry occurs in the least profitable market

$a_B < a_A$ (D_A = 20, D_B = 20, c_A = 10)

		d = 1	d = 0.8	d = 0.6	d = 0.4	d = 0.2
c_B=15	w^m	0.822	1.018	1.221	1.233	1.220
	$\hat{w}$ ($n \to \infty$)	0.615	0.783	0.977	1.206	1.488
	$\hat{w}$ (n=2)	0.540	0.715	0.915	1.151	1.444
c_B=12.5	w^m	0.935	1.112	1.142	1.146	1.127
	$\hat{w}$ ($n \to \infty$)	0.724	0.899	1.093	1.308	1.559
	$\hat{w}$ (n=2)	0.642	0.828	1.030	1.255	1.519

1.c. Entry occurs in the most profitable market

$a_B > a_A$ (D_A = 20, D_B = 20, c_B = 10)

		d = 1	d = 0.8	d = 0.6	d = 0.4	d = 0.2
c_A=15	w^m	0.822	1.018	1.221	1.233	1.220
	$\hat{w}$ ($n \to \infty$)	0.909	1.085	1.268	1.457	1.660
	$\hat{w}$ (n=2)	0.815	1.010	1.208	1.409	1.626
c_A=12.5	w^m	0.935	1.112	1.142	1.146	1.127
	$\hat{w}$ ($n \to \infty$)	0.846	1.023	1.211	1.410	1.628
	$\hat{w}$ (n=2)	0.756	0.950	1.149	1.359	1.592

With cost substitutability and fixed cost subadditivity, the entrant firms have a lower marginal cost than the multi-product firm. Thus, entry in market B produces an output shift towards more efficient firms. However, that shift goes in the opposite direction in market A. It is worth noting one of the properties of $\tilde{C}_M$: each product's average incremental cost is decreasing in own output. This means, contrary to the cost complementarity case, that the multi-product firm has something to gain by shifting production from market B - where average incremental cost has gone up - to market A - where average incremental cost has gone down. In fact, the output reallocation by the multi-product firm is such that total output not only raises in market B but also in market A. Therefore, the output reallocation within and across markets results in a welfare gain under cost substitutability and fixed cost subadditivity.

Table 2 summarizes the welfare effects of the two competition enhancing measures analyzed in the presence of scope economies. The move to the corporate divestiture regime entails a welfare reduction under cost complementarity. On the other hand, the move to the competitive regime supposes a welfare gain under cost substitutability and fixed cost subadditivity. The remaining two cases yield ambiguous effects on welfare. As the taxonomy in Table 2 shows, more competition entails a welfare loss under some particular conditions. Such a theoretical result would appear more useful were we capable of identifying some industries which fulfil those conditions.

TABLE 2
A taxonomy on the welfare effects of more competition

		Type of economies of scope	
		With cost complement.	*With cost substitutability*
Competition	*Divestiture*	–	ambiguous
Enhancing measures	*Entry*	ambiguous	+

The rail sector is a suitable industry. Some authors (Preston and Nash, 1996, and Cantos-Sánchez, 2000, 2001) have found evidence on cost substitutability between the rail operating costs (excluding infrastructure costs) of passenger and freight operations for the European industry.[9] According to our taxonomy, the social desirability of keeping both operations together in the same firm will depend on the magnitude of fixed cost subadditivity. If it compensates for the inefficiency in marginal cost then corporate divestiture supposes a welfare improvement. On the other hand, and for the U.S. rail sector, Ivaldi and McCullogh (2000) show the existence of cost complementarity among different freight operations as e.g. between general and intermodal freight. Our

[9] At the average data of the sample, Cantos-Sánchez (2001) obtains that a 1 % increase in freight traffic increases the marginal cost of passenger transport by 0.07 %, whereas a 1 % increase in passenger traffic increases the marginal cost of freight transport by 0.013 %.

conclusions suggest that the multi-product freight companies should not be separated into single-product companies. Although there are no econometric results focusing exclusively on passenger operations, Kessides and Willig (1995) offer a number of arguments that suggest the presence of scope economies resulting from the common costs of rail passenger operations. To sum up, the evidence indicates there is cost substitutability between passenger and freight operations at an aggregate level, and yet there is cost complementarity within different types of passenger or freight operations[10].

Concerning the move to the competitive regime, suppose a railway company that integrates several freight operations. The long-distance service unit, typically characterized by intermodal or bulk traffic, competes with the port system while the medium and short-distance service unit competes with road transport firms. Both the port and the trucking sectors have traditionally been strongly regulated sectors. Thus, road transport firms were required a license to operate. In Spain, the removal of such licenses has led to a decrease in road prices and to an output increase. In light of our results, the magnitude of cost complementarities should be evaluated in order to assess whether the deregulatory measures taken may bring about a welfare gain.

Economies of scope have been found to exist in the telecommunications industry. A number of recent papers, as e.g. Gabel and Kennet (1994) and Banker *et al.* (1998), establish that the generation of multiple outputs leads to a reduction in average unit costs, that is, the joint cost function is subadditive. In fact, network industries, most of them regulated over the years, should be particularly looked at since they are thought to be characterized by joint economies of scale. Network industries combine both the opportunity for productive efficiency and the potential for market power. Thus, and according to our analysis, electric power industries also merit particular attention. Other industries liable of study are the banking, insurance, airlines and auto-

[10] Our analysis can alternatively be interpreted as an explanation to mergers, that is, by taking the move from the corporate divestiture to the diversified regime. In fact, Ivaldi and McCullogh (2000) note that the merger process in the Class I freight railroads is partially explained by the existence of cost complementarity among different rail freight activities. Very possibly, cost complementarity appears as one of the reasons behind the merger process in the British rail freight sector. As noted in footnote 4, the privatization and deregulation measures adopted in 1995 resulted in three rail freight companies which have just merged together.

mobile producers industries, only to mention a few (see Panzar, 1989, specifically pages 46-55).

5. Concluding remarks

The paper suggests, by means of a class of examples, how watchful competition analysis should be in the presence of economies of scope. The main lesson from our analysis is that competition authorities should not solely evaluate particular deregulatory measures on the basis of the number of independent firms and concentration indices. We have investigated the welfare effects of two competition enhancing measures both leading to more firms in the market: the divestiture of a multi-product firm and entry of single-product firms in one market. In this context, more firms does not necessarily guarantee welfare gains. With scope economies, markets are effectively connected through the variable cost component of multi-product firms. In such a setting, a deregulatory measure designed to promote competition in a particular market can affect competition in other markets.

Attention has been drawn to a number of elements that should be taken into account in competition policy analysis. These elements are summarized in a taxonomy which combines the particular way of introducing competition as well as the type and the size of economies of scope. There are certainly other elements at play that affect multi-product firms and have been disregarded. Among these, efficiency gains by an internal reorganization, the possibility of regulated prices and cross-subsidization, and changes in efficiency due to the competitive pressure derived by entry. More specifically, welfare is more likely to be reduced, in a move to the divestiture regime, either for low values of cost substitutability and a high degree of product differentiation, or for cost complementarity. Entry of single-product firms in one market will likely result in a welfare reduction for large values of cost complementarity and the lower the degree of product differentiation. Taking the necessary qualifications, the analysis undertaken is applicable to high-tech and network industries such as transport, telecommunications and electric power industries. In fact, the multi-product nature of firms is intrinsic to most real world industries. The interesting results obtained are an invitation for researchers to direct their attention to these regulatory aspects in industrial economics. Possible extensions are to introduce some of the aforementioned elements as well as to consider different objective functions for firms, to include the possibility of creating strategic alliances and to introduce $R\&D$ decisions.

Appendix A1

A1.1 Proof of Proposition 1

We want to find under which conditions there is a positive difference in social welfare when we compare the diversified regime with the corporate divestiture regime, that is, when $SW^{df} - SW^{dv} > 0$. Under cost complementarity, the increment in the multi-product firm's profits is:

$$\begin{aligned}\Delta\Pi_M &= \Pi_M^{df} - \Pi_M^{dv} = b_A\left((q_{MA}^{df})^2 - (q_{MA}^{dv})^2\right) + \\ &\quad b_B\left((q_{MB}^{df})^2 - (q_{MB}^{dv})^2\right) - wq_{MA}^{df}q_{MB}^{df}\end{aligned}$$

which after substitution of equilibrium quantities is equal to:

$$\begin{aligned}\Delta\Pi_M &= \frac{w}{D} \times [(2b_B - d)(2b_A - d)(2b_B + d)^2(2b_A + d)^2\Phi_{BA}a_Ba_A + \\ &\quad +2wb_A(2b_A + d)^2\Phi_B a_B^2 \\ &\quad +2wb_B(2b_B + d)^2\Phi_A a_A^2]\end{aligned}$$

where $D \equiv (2b_B+d)^2(2b_A+d)^2((4b_B^2-d^2)(4b_A^2-d^2)-4b_Bb_Aw^2)^2 > 0$, $\Phi_{BA} \equiv 16b_B^2b_A^2 - d^4 - 4b_Bb_Aw^2$, $\Phi_B \equiv (4b_B^2 - d^2)(8b_B^2b_A^2 + 2b_A^2d^2 - d^4) - 8b_B^3b_Aw^2$ and finally $\Phi_A \equiv (4b_A^2 - d^2)(8b_B^2b_A^2 + 2b_B^2d^2 - d^4) - 8b_A^3b_Bw^2$. The three coefficients Φ_{BA}, Φ_B, and Φ_A are positive for some bounds on w. However, the second order and stability conditions for a maximum in the diversified regime (a positive denominator in the equilibrium outputs) are more restrictive than those bounds. Therefore, $\Delta\Pi_M > 0$ and the multi-product firm earns more profits when there are scope economies with cost complementarities.

The difference in aggregate profits is ΔAP

$$AP^{df} - AP^{dv} = \Delta\Pi_M + b_A\left((q_{SA}^{df})^2 - (q_{SA}^{dv})^2\right) + b_B\left((q_{SB}^{df})^2 - (q_{SB}^{dv})^2\right)$$

which is equal to,

$$\begin{aligned}\Delta AP &= \frac{w}{D}[(2b_B + d)^2(2b_A + d)^2\Omega_{BA}a_Ba_p + wb_A(2b_A + d)^2\Omega_B a_B^2 + \\ &\quad +wb_B(2b_B + d)^2\Omega_A a_A^2]\end{aligned}$$

where $\Omega_{BA} \equiv (2b_A-d)^2(2b_B-d)^2(4b_Bb_A-d^2)-4b_Bb_A(4b_Bb_A-4d(b_B+b_A)+3d^2)w^2$, $\Omega_A \equiv (2b_A-d)^2(2b_A+d)(4b_B^2(2b_A-d)+d^2(2b_A+3d)) -4b_Bb_A(4b_A^2-4b_Ad-d^2)w^2$, $\Omega_B \equiv (2b_B-d)^2(2b_B+d)(4b_A^2(2b_B-d)+d^2(2b_B+3d))$ $-4b_Bb_A(4b_B^2-4b_Bd-d^2)w^2$.

As above, the restrictions on w that make Ω_{BA}, Ω_A and Ω_B positive are weaker than the second order and stability conditions for a maximum. Therefore, aggregate profits are greater when there are scope economies with cost complementarities.

Finally, the consumer surplus increment is,

$$\begin{aligned}\Delta CS &= CS^{df}-CS^{dv}= \\ &\frac{1}{2}\{b_A\left[(q_{MA}^{df})^2+(q_{SA}^{df})^2-(q_{MA}^{dv})^2-(q_{SA}^{dv})^2\right]+ \\ &b_B\left[(q_{MB}^{df})^2+(q_{SB}^{df})^2-(q_{MB}^{dv})^2-(q_{SB}^{dv})^2\right]\}+ \\ &d\left(q_{MA}^{df}q_{SA}^{df}+q_{MB}^{df}q_{SB}^{df}-q_{MA}^{dv}q_{SA}^{dv}-q_{MB}^{dv}q_{SB}^{dv}\right)\end{aligned}$$

which is equal to,

$$\Delta CS = \frac{w}{2D}[2(2b_B+d)^2(2b_A+d)^2\Theta_{BA}a_Ba_p+wb_A(2b_A+d)^2\Theta_Ba_B^2+ \\ +wb_B(2b_B+d)^2\Theta_Aa_A^2]$$

where $\Theta_{BA} \equiv (2b_A-d)^2(2b_B-d)^2(4b_Bb_A+3b_Bd+3b_Ad+2d^2)-4b_Bb_Ad(b_B+b_A-d)w^2$, $\Theta_B \equiv (2b_B-d)^2(2b_B+d)(24b_A^2b_B+20b_Ad-10b_Bd^2-7d^3))$ $-4b_Bb_A(4b_B^2-4b_Bd-d^2)w^2$, $\Theta_A \equiv (2b_A-d)^2(2b_A+d)(24b_B^2b_A+20b_Bd-10b_Ad^2-7d^3))$ $-4b_Bb_A(4b_A^2-4b_Ad-d^2)w^2$, and the three of them are positive when the second order and stability conditions for a maximum are satisfied.

A1.2 Proof of Proposition 2

We prove that $SW^{df}-SW^{dv}>0$ for a collection of examples in the cost substitutability case.

Assume symmetric markets, that is $a_A=a_B=a$ and $b_A=b_B=1$. For the sake of the proof we find the conditions under which the diversified regime (df) is preferred to the corporate divestiture (dv) regime in welfare terms. i.e. $SW^{df}(w)-SW^{dv}(w)>0$. That increment can be written as the sum of one term, which is a function of w, denoted by $VSW^{df}(w)$– $VSW^{dv}(w)$, and another which takes into account the difference in fixed costs, denoted by $\Delta F \equiv F_A+F_B-F_{AB}$. Then, checking that $SW^{df}(w)-SW^{dv}(w)>0$.is equivalent to checking that

$\Delta F > VSW^{dv}(w) - VSW^{df}(w)$. ΔF is positive by the subadditivity of the multi-product cost function and $VSW^{dv}(w) - VSW^{df}(w) = a^2 Z(w)$, where $Z(w)$ is,

$$Z(w) \equiv \frac{w[(2-d)^2(2+d)(4+d)+(12-4d-7d^2)w]}{(2+d)^2(4-d^2+2w)^2}$$

$Z(w)$ satisfies $Z(0) = 0$, $Z(w) > 0$ for all $w > 0$. Then, whenever $\frac{\Delta F}{a^2} > (<) Z(w)$ more competition, i.e. divestiture, is welfare reducing (improving). Therefore, it is important not only to find whether $Z(w)$ is increasing with w and at which speed but also to find the lowest upper bound on w which may limit the $Z(w)$ highest value.

We begin by finding $w^{\text{máx}}$, the lower upper bound on w. Firstly, the positive quantities restriction is given by the positiveness of $min\{q^{df}_{MA}(w), q^{df}_{MB}(w)\}$ which implies the following upper bounds on w, $w < \text{mín}\left\{\frac{a_A(2b_A-d)(2b_B+d)}{2a_Ab_A}, \frac{a_B(2b_B-d)(2b_A+d)}{2a_Bb_B}\right\}$. Secondly, the second order condition is $w < \left(\frac{(4b_A^2-d^2)(4b_B^2-d^2)}{4b_Ab_B}\right)^{\frac{1}{2}}$. Note that, by the symmetry assumption, these latter conditions become $w < 2 - \frac{d^2}{2}$. Note also that, since the multi-product firms enjoys scope economies with cost substitutability, it must be the case that $\Delta F > wq^{df}_{MA}(w)q^{df}_{MB}(w)$, which imposes an upper bound on w, which can be written as $\frac{\Delta F}{a^2} > \frac{w(2-d)^2}{(4-d^2+2w)^2}$. We denote the right hand side of that condition by $R(w)$. It satisfies the following properties: a)$R(0) = 0$, b)$R(w) > 0$ for all $w > 0$, c) $R(w)$ attains a maximum at $w = 2 - \frac{d^2}{2}$,at the level $R(w = 2 - \frac{d^2}{2}) = \frac{(2-d)}{8(2+d)}$, and finally d) $Z(w) > R(w)$ for all $w > 0$.

In order to obtain $w^{\text{máx}}$, $\bar{w}(\frac{\Delta F}{a^2})$ is defined as the maximum level of cost substitutability compatible with the existence of scope economies for each $\frac{\Delta F}{a^2}$, that is, $\bar{w}(\frac{\Delta F}{a^2})$ is the lowest root of the concave second order equation on w, defined by $\frac{\Delta F}{a^2} = \frac{w(2-d)^2}{(4-d^2+2w)^2}$. Also, $\bar{w}(\frac{\Delta F}{a^2})$ is increasing with $\frac{\Delta F}{a^2}$ and exists as long as $\frac{\Delta F}{a^2} \in \left[0, \frac{(2-d)}{8(2+d)}\right]$. Therefore, $w^{\text{máx}} \equiv \text{mín}\{\bar{w}(\frac{\Delta F}{a^2}), 2 - \frac{d^2}{2}\}$ where $w^{\text{máx}} = \bar{w}(\frac{\Delta F}{a^2})$ as long as $0 < \frac{\Delta F}{a^2} \leq \frac{2-d}{8(2+d)}$, otherwise $w^{\text{máx}} = 2 - \frac{d^2}{2}$.

Next we prove that $\frac{dZ(w)}{dw} = Z'(w) > 0$ for $w \in (0, w^{\text{máx}}]$. The sign$[Z'(w)]$ = sign$[(2-d)^2(2+d)(4+d)+4(1+d)(2-3d)w]$, which is positive. Simple algebra shows that for $0 < d \leq \frac{2}{3}$ the sign is positive and for $\frac{2}{3} < d < 1$, it is shown that $(2-d)^2(2+d)(4+d)+ 4(1+d)(2-3d)w^{\text{máx}} > 0$ as long as $(2-d)(4+d) > 2(1+d)(3d-2)$ which is equivalent to $(12-4d-7d^2) > 0$ and is the case. Since $Z(w)$ is increasing for $w \in (0,\, w^{\text{máx}})$, the largest value it can take is $Z(w = 2 - \frac{d^2}{2}) = \frac{28-8d-9d^2}{16(2+d)^2}$, and therefore for every $\frac{\Delta F}{a^2} > \frac{28-8d-9d^2}{16(2+d)^2}$, it is satisfied that $SW^{df} - SW^{dv} > 0$, for $w \in (0,\, w^{\text{máx}})$. This is part a.2) of the result below. For $0 < \frac{\Delta F}{a^2} \leq \frac{28-8d-9d^2}{16(2+d)^2}$, we proceed by defining $w^*(\frac{\Delta F}{a^2})$ as the level of cost substitutability that makes the social welfare coincide for both the diversified regime and the corporate divestiture regime for each $\frac{\Delta F}{a^2}$. Then $w^*(\frac{\Delta F}{a^2})$ is the lowest root of the concave second order equation on w, defined by $\frac{\Delta F}{a^2} = \frac{w[(2-d)^2(2+d)(4+d)+(12-4d-7d^2)w]}{(2+d)^2(4-d^2+2w)^2}$. It is also increasing with $\frac{\Delta F}{a^2}$ and it satisfies $w^*(\frac{\Delta F}{a^2}) < \bar{w}(\frac{\Delta F}{a^2})$ whenever the latter exists since $Z(w) > R(w)$. Therefore, we can partition the interval $(0, w^{\text{máx}}]$ in two subsets, $(0,\, w^*)$ and $[\, w^*, w^{\text{máx}}]$ for every $\frac{\Delta F}{a^2} \in (0,\, \frac{28-8d-9d^2}{16(2+d)^2}]$. The former is the set of w that implies $SW^{df} - SW^{dv} > 0$, part a.1) in the next result. The latter set implies the opposite $SW^{df} - SW^{dv} < 0$, part b) of the result. Note that parts b.1) and b.2) take into account whether $w = \bar{w}(\frac{\Delta F}{a^2})$ or $w = 2 - \frac{d^2}{2}$ is the $w^{\text{máx}}$. Therefore, the next result follows:

a) Divestiture is welfare reducing if,

a.1) either $0<\frac{\Delta F}{a^2} \leq \frac{28-8d-9d^2}{16(2+d)^2}$ *and* $w \in (0, w^*(\frac{\Delta F}{a^2}))$,

a.2) or $\frac{\Delta F}{a^2} > \frac{28-8d-9d^2}{16(2+d)^2}$ *and* $w \in (0, 2 - \frac{d^2}{2}]$.

b) Divestiture is welfare improving if

b.1) either $0< \frac{\Delta F}{a^2} \leq \frac{2-d}{8(2+d)}$ *and* $w \in [w^*(\frac{\Delta F}{a^2}),\, \bar{w}(\frac{\Delta F}{a^2})]$

b.2) or $\frac{2-d}{8(2+d)} < \frac{\Delta F}{a^2} \leq \frac{28-8d-9d^2}{16(2+d)^2}$ *and* $w \in [w^*(\frac{\Delta F}{a^2}), 2 - \frac{d^2}{2}]$.

Finally note that $\frac{28-8d-9d^2}{16(2+d)^2}$ is decreasing with d, and is equal to $\frac{7}{16}$ for $d = 0$. It establishes a sufficient condition on $\frac{\Delta F}{a^2}$ for more competition to be welfare improving regardless of the degree of product differentiation and the size of cost substitutability.

A1.3 Proof of Proposition 3

Note that $SW^e(n) - SW^{df} = SW^e(n) - SW^e(1)$. To prove that entry is welfare improving amounts to proving that $\frac{\partial SW^e(n)}{\partial n} > 0$ for $n \geq 2$.

We prove it for $\tilde{C}_M$. The sign of this derivative is given by the sign of the following polynomial:

$$\begin{aligned} & a_A w d[(2b_A - d)^2(b_B^2(4b_A + d) + (16b_A b_B^2 + 7b_B^2 d - 3b_A d^2 - d^3)n) - \\ & \quad b_A b_B(12b_A b_B((12b_A - 7d)n - d)w^2] + \\ & \quad a_B[(4b_A^2 - d^2)^2((12b_B^3 - 7b_B^2 d - b_B d^2 + d^3)n - b_B^2 d) - \\ & \qquad b_A(4b_A^2[(24b_B^2 - 5b_B d - 3d^2)n + b_B d] - \\ & \qquad d^2[(24b_B^2 - 7b_B d - d^2)n - b_B d])w^2 + \\ & \qquad 12b_A^2 b_B n w^4] \end{aligned} \quad \text{[A1.1]}$$

Suppose $b_A = b_B = 1$. The bounds on w become $w \in (0, \min\{\frac{a_B(2+d)(1+n-nd)}{a_A(1+n)}, \frac{a_A(4-d^2)}{2a_B}\})$ (the bounds for positive quantities are more restrictive than the second order and stability conditions.). The a_A coefficient in $(A1,1)$ is decreasing in w. It suffices to take the greatest value for w and check that the coefficient is positive. For the sake of the proof let $w = \frac{(4-d^2)(2-d)}{2}$, (which ensures $\frac{\partial q_{SB}}{\partial a_B} > 0$ and is a less restrictive value than $\min\{\frac{a_B(2+d)(1+n-nd)}{a_A(1+n)}, \frac{a_A(4-d^2)}{2a_B}\}$) which upon substitution yields a positive value for the above mentioned coefficient.

Similarly, the a_B coefficient is decreasing in w. Two cases must be distinguished to establish whether it is positive. Let $r = \frac{a_B}{a_A}$. It is easy to check that for $0 < r^2 < \frac{(2-d)(1+n)}{2(1+n-nd)}$ the binding value for w is $\frac{r(2+d)(1+n-nd)}{(1+n)}$. Substitute the a_B coefficient for $w = \frac{r(2+d)(1+n-nd)}{(1+n)}$ which yields a convex polynomial of degree four in r. The smallest root of this polynomial lies above $\frac{(2-d)(1+n)}{2(1+n-nd)}$ for $n \geq 3$, and hence the a_B coefficient is positive for $n \geq 3$. Finally, substitute $(A1,1)$ for $n = 2$ and $w = \frac{r(2+d)(1+n-nd)}{(1+n)}$. Algebraic computations show that it is indeed positive for $0 < r^2 < \frac{(2-d)(1+n)}{2(1+n-nd)}$.

On the other hand, for $\frac{(2-d)(1+n)}{2(1+n-nd)} \leq r^2 < \infty$, the binding value for w is $\frac{(4-d^2)}{2r}$. Since the a_B coefficient is not positive for all $d \in (0,1)$ and for all $2 \leq n$, the strategy of the proof is to establish the sign of $(A1{,}1)$ by letting $w = \frac{(4-d^2)}{2r}$, i.e. the sign of

$$\begin{aligned}&(4-d^2)^2(d^2+(24-12d+d^2)n)+\\&2(2-d)(-192n+d(8-8d-4d^2-d^3+\\&(8+88d-4d^2-7d^3+d^4)n))r^2+\\&8(4-d^2)(-d+(12-7d-d^2+d^3)n)r^4\end{aligned} \qquad [A1.2]$$

We prove first that it is increasing with r^2. Since the first derivative is positive for the smallest r^2, $i.e. \frac{(2-d)(1+n)}{2(1+n-nd)}$, then $(A1{,}2)$ is increasing with r^2 for all d and n. Substituting for $r^2 = \frac{(2-d)(1+n)}{2(1+n-nd)}$ in $(A1{,}2)$ we find that it is positive whenever $(1-(4+6d+d^2)n+(7+6d+2d^2)n^2)$ which is the case for $n \geq 2$.

Appendix A2: Properties of the cost functions

When firms produce more than one good, and when total costs are affected by the mix of goods produced as well as by the volume of total production, the appropriate measure of scale economies is $S = \frac{C(q)}{qMC(q)}$, where q is the vector of all outputs and it has n-components, and $MC(q)$ is the marginal cost vector. This measure implies that the mix of products is kept constant. Economies of scale are increasing, constant or decreasing for S greater than, equal to or less than one. Hence, for the cost function $\hat{C}_M$ we have that $\widehat{MC}_{q_{MA}} = c_A - wq_{MB}$ (which does not depend on q_{MA}), $\widehat{MC}_{q_{MB}} = c_B - wq_{MA}$ (which does not depend on q_{MB}). The measure S is equal to

$$\frac{c_Aq_{MA}+c_Bq_{MB}-wq_{MA}q_{MB}}{(q_{MA},q_{MB})\begin{pmatrix}c_A-wq_{MB}\\c_B-wq_{MA}\end{pmatrix}} = \frac{c_Aq_{MA}+c_Bq_{MB}-wq_{MA}q_{MB}}{c_Aq_{MA}+c_Bq_{MB}-2wq_{MA}q_{MB}} > 1$$

The cost function $\hat{C}_M$ exhibits increasing economies of scale. On the other hand, $\widetilde{MC}_{q_{MA}} = c_A + wq_{MB}$, and $\widetilde{MC}_{q_{MB}} = c_B + wq_{MA}$. We have:

$$\begin{aligned}S \;=\; &\frac{c_Aq_{MA}+c_Bq_{MB}+wq_{MA}q_{MB}+F_{AB}}{(q_{MA},q_{MB})\begin{pmatrix}c_A+wq_{MB}\\c_B+wq_{MA}\end{pmatrix}} =\\&\frac{c_Aq_{MA}+c_Bq_{MB}+wq_{MA}q_{MB}+F_{AB}}{c_Aq_{MA}+c_Bq_{MB}+2wq_{MA}q_{MB}}\end{aligned}$$

Consequently, $S > 1$ as long as $F_{AB} > wq_{MA}q_{MB}$.

Product specific scale economies is a measure that takes into account the effect on costs brought on by changes in the mix of goods produced. It is defined as $S_i = \frac{AIC(q_i)}{MC_i}$, where $AIC(q_i)$ denotes the average incremental cost of producing good i, i.e. $\frac{C(q)-C(q_1,q_2,...q_{i-1},0,q_{i+1},...q_n)}{q_i}$. Thus, $\widehat{AIC}_{q_{MA}} = \frac{c_A q_{MA}+c_B q_{MB}-w q_{MA} q_{MB}-c_B q_{MB}}{q_{MA}} = c_A - w q_{MB}$ which is equal to $\widehat{MC}_{q_{MA}}$. The same happens for q_{MB}. It follows that the cost function $\hat{C}_M$ exhibits constant product specific scale economies. Concerning $\tilde{C}_M$, the average incremental cost for good A is

$$\widetilde{AIC}_{q_{MA}} = \frac{c_A q_{MA} + c_B q_{MB} + w q_{MA} q_{MB} + F_{AB} - c_B q_{MB} - F_B}{q_{MA}} = c_A + w q_{MB} + \frac{F_{AB} - F_B}{q_{MA}}$$

which is decreasing in q_{MA}. The measure $S_{q_{MA}}$ is equal to

$$\frac{c_A + w q_{MB} + \frac{F_{AB}-F_B}{q_{MA}}}{c_A + w q_{MB}} = 1 + \frac{F_{AB} - F_B}{c_A q_{MA} + w q_{MA} q_{MB}} > 1$$

The same happens for good B and we conclude that $\tilde{C}_M$ exhibits increasing product specific scale economies.

Appendix A3: Technical bounds on w.

There are three sources for technical restrictions on w :

a) the second order and stability conditions for a maximum,

b) interior solutions in quantities, and

c) restrictions from the specific cost function.

In particular, a) Second order and stability conditions come from the second partial derivatives. It is known that stability conditions either coincide or are stronger than second order conditions. The equilibrium is stable if the determinant of the Hessian matrix is positive, and all of the elements in the diagonal are negative. The Hessian matrix is

$$H = \begin{pmatrix} -2b_A & w & -d & 0 \\ w & -2b_B & 0 & -d \\ -d & 0 & -2b_A & 0 \\ 0 & -d & 0 & -2b_B \end{pmatrix}$$

where $|H| = (4b_A^2 - d^2)(4b_B^2 - d^2) - 4b_A b_B w^2 > 0$. Note that this expression is the denominator of [7]-[8] and it is the same for both specifications of the cost function. Similarly, in the competitive regime the stability conditions coincide with the denominator in [9]-[12]. Finally, note that the former is more restrictive for any $n \geq 2$.

b) Interior solutions in quantities are ensured as long as the minimum of the numerators of all the equilibrium quantities be positive. Note that, for the cost function $\hat{C}_M$, single-product equilibrium outputs are smaller than those of the multi-product firm for both the diversified and the competitive regimes. The opposite happens for the cost function $\tilde{C}_M$. Finally note that when moving from the diversified regime to the competitive regime under $\tilde{C}_M$, the output of the multi-product firm in market B decreases while its output in market A increases, therefore interior solutions in quantities follow as long as the minimum of the numerators in q^e_{MB} and q^{df}_{MA} be positive. Similarly under $\hat{C}_M$, interior solutions in quantities follow as long as the minimum of the numerators in q^e_{SB} and q^{df}_{SA} be positive.

c) Restrictions from the specific cost function. For $\hat{C}_M$ multi-product marginal costs must be positive. Since the multi-product firm outputs decrease with entry it must be the case that $c_A - wq^{df}_{MB} > 0$ and $c_B - wq^{df}_{MA} > 0$. For $\tilde{C}_M$ the cost function must be sufficiently subadditive in fixed costs to have economies of scope. For the diversified regime $w < (F_A + F_B - F_{AB})/(q^{dv}_{MA} q^{dv}_{MB})$.

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Resumen

En este trabajo se examinan los cambios en el bienestar social derivados de la introducción de mayor competencia en mercados tecnológicamente relacionados. Se desarrolla un modelo con dos mercados donde en cada uno de ellos opera un duopolio de producto diferenciado formado por una empresa multiproducto y una empresa uniproducto. En este contexto se consideran dos medidas que potencian, a priori, la competencia: la separación de la empresa multiproducto en dos empresas uniproducto y, alternativamente, la entrada de empresas uniproducto en uno de los mercados. Los resultados indican que mayor competencia podría llevar a una reducción del bienestar social. Este modelo, por tanto, destaca la relevancia del tipo y magnitud de las economías de alcance, la forma de introducir mayor competencia y el grado de diferenciación de producto en los análisis de bienestar.

Palabras clave: Complementariedad/sustituibilidad en costes, subaditividad en costes fijos, bienestar social.

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